Engineering controlled quantum systems

Alain Sarlette

INRIA (QUANTIC team)

Wed. 26th November, 2014

Quantum IT is becoming a reality today.

Algorithms (80s)

- “BB84” algorithm: quantum “no-cloning” theorem guarantees impossibility to spy on communication (physically $\neq$ NP-hard)
- “Shor” algorithm for polynomial-time integer factorization

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Experimental components (90s): controlling one quantum degree of freedom

excitation level of single atoms [$\mathbb{C}^2$ Hilbert space]
position of trapped ions
EM standing wave: circuit / μwave cavity [$\mathbb{C}^{\mathbb{N}}$ Hilbert space]

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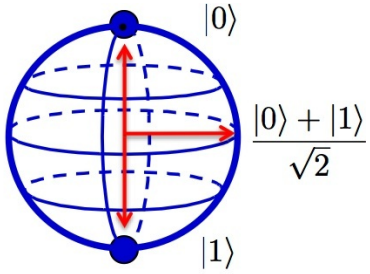
First commercial products (00s)

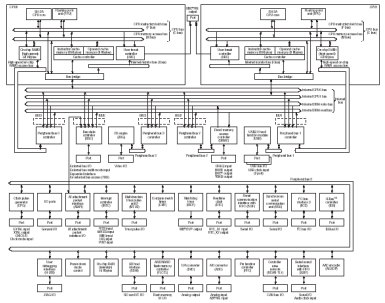
company IDQ (random key generation & distribution)

Nature 24/04/2014 "China begins work on super-secure network as ?real-world? trial successfully sends quantum keys and data."

Next step: from components / software to hardware systems



$\longrightarrow$

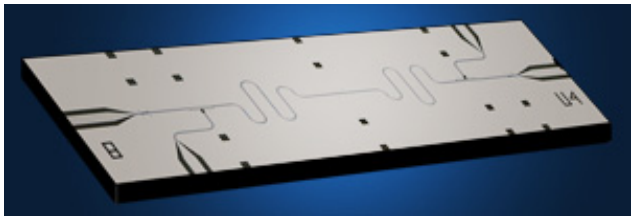


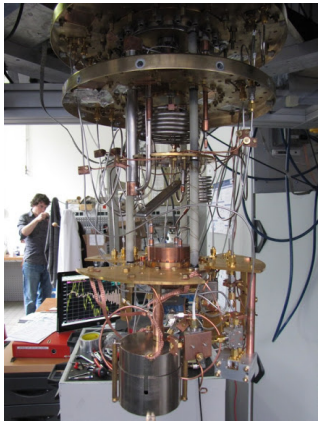
Develop

- proper integration of interacting active components
  more than "keep your state" goal of current commercial products
  more than the 4 or 5 components in current "quantum chips"
- robustness to disturbances, error correction strategies
  (software error correction already exists)
- scalability of physical behavior & of design methods
- improved components towards this goal

Why is the "classical" systems approach insufficient?

- In the quantum model, action and measurement are not separated: taking measurement = interacting with system = applying action
- Interconnecting quantum systems changes their state space (see later in this talk)
- Extreme time and physical scales (μseconds, *m*Kelvin noise,...)





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Outline

1. isolated quantum systems $\sim$ classical model
2. open, interacting quantum systems $\neq$ classical model
3. two types of "feedback", proved experimentally

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Isolated quantum systems just follow particular classical dynamics

$$\frac{d}{dt}x = A\,x$$

$$\frac{d}{dt}x = A(u)\,x$$

u control parameter

e.g. ball on u-shaped surface

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u control parameter

e.g. ball on u-shaped surface

$$\frac{d}{dt}|\psi\rangle = -i\,H\,|\psi\rangle$$

$$\frac{d}{dt}|\psi\rangle = -i\,H(u)\,|\psi\rangle$$

u control parameter

e.g. spin in u-electric field

Notation: physicists prefer Greek to Latin...

$|\psi\rangle \in \mathbb{C}^N$ denotes a complex vector

$\langle\psi| = |\psi\rangle^\dagger$ is its Hermitian transpose (dual space)

$\langle\psi_1|\psi_2\rangle = \langle\psi_1|\,|\psi_2\rangle$ is the Hermitian scalar product

Isolated quantum systems just follow particular classical dynamics

$$\frac{d}{dt}|\psi\rangle \;=\; -i\,H(u)\,|\psi\rangle$$

State $|\psi\rangle \in \mathbb{C}^N \quad , \quad \langle\psi|\psi\rangle = 1$ (complex sphere)
"wave function" or "probability density"

Hamiltonian H hermitian such that iH implies a (complex) rotation.

Caveats:

- Hamiltonian dynamics, not valid if the system interacts (dissipation **or measurement**)
- Global phase is “irrelevant”: $|\psi\rangle \sim e^{i\phi}\,|\psi\rangle$. Actual system state is on complex projective space: $\rho = |\psi\rangle\langle\psi|$.
- open-loop Hamiltonian dynamics **cannot stabilize**

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   System-System interaction
   System-OutsideWorld Interaction

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Classical systems in interaction evolve on the **cartesian product** of individual state spaces

System 1: $x_a \in \mathcal{S}_a$ with orthonormal basis $\mathbf{e}_{1a},\ \mathbf{e}_{2a},\ \ldots,\ \mathbf{e}_{Na}$

System 2: $x_b \in \mathcal{S}_b$ with orthonormal basis $\mathbf{e}_{1b},\ \mathbf{e}_{2b},\ \ldots,\ \mathbf{e}_{Nb}$

Systems 1 & 2 interacting: $x \in \mathcal{S}_a \times \mathcal{S}_b$ with orthonormal basis

$$\mathbf{e}_{1a} \times \mathbf{0}_b \ , \quad \mathbf{e}_{2a} \times \mathbf{0}_b \ , \ \ldots \ , \quad \mathbf{e}_{Na} \times \mathbf{0}_b \ , \quad \mathbf{0}_a \times \mathbf{e}_{1b} \ , \ \ldots \ , \quad \mathbf{0}_a \times \mathbf{e}_{Nb}.$$

$\Rightarrow$ dimension $N_a + N_b$

Quantum systems in interaction evolve on the **tensor product** of individual state spaces

System 1: $|\psi_a\rangle \in \mathcal{H}_a$ with orthonormal basis $|1_a\rangle,\ |2_a\rangle,\ \ldots,\ |N_a\rangle$

System 2: $|\psi_b\rangle \in \mathcal{H}_b$ with orthonormal basis $|1_b\rangle,\ |2_b\rangle,\ \ldots,\ |N_b\rangle$

Systems 1 & 2 interacting: $|\psi\rangle \in \mathcal{H}_a \otimes \mathcal{H}_b$ with orthonormal basis

$$
\begin{array}{llll}
|1_a\rangle \otimes |1_b\rangle\ , & |1_a\rangle \otimes |2_b\rangle\ , & \ldots\ , & |1_a\rangle \otimes |N_b\rangle\ , \\
|2_a\rangle \otimes |1_b\rangle\ , & |2_a\rangle \otimes |2_b\rangle\ , & \ldots\ , & |2_a\rangle \otimes |N_b\rangle\ , \\
\ldots & & & \\
|N_a\rangle \otimes |1_b\rangle\ , & |N_a\rangle \otimes |2_b\rangle\ , & \ldots\ , & |N_a\rangle \otimes |N_b\rangle\ .
\end{array}
$$

$\Rightarrow$ dimension $N_a * N_b$

The dynamics of interaction is richer for quantum systems than for classical systems

N interacting D-dimensional systems

classical: cartesian product state $D * N$

quantum: tensor product state D^N

joint probability: $f(x_1, x_2, ...)$ is also D^N - dimensional

$\Rightarrow$ Quantum systems "are probabilistic"... **and a bit more:**

The dynamics of interaction is richer for quantum systems than for classical systems

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joint probability: $f(x_1, x_2, ...)$ is also D^N - dimensional

$\Rightarrow$ Quantum systems "are probabilistic"... **and a bit more:**

States $|\psi\rangle$ that cannot be written $|\psi\rangle = |\psi_a\rangle \otimes |\psi_b\rangle$ are **entangled**.

- Bell state $\frac{1}{\sqrt{2}}\left(|0_a\rangle|0_b\rangle + |1_a\rangle|1_b\rangle\right) = \frac{1}{\sqrt{2}}\left(|x_{+a}\rangle|x_{+b}\rangle + |x_{-a}\rangle|x_{-b}\rangle\right)$
  e.g. where $|x_+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|x_-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Schrödinger cat $\frac{1}{\sqrt{2}}(|\text{dead}\rangle|\text{photon shot}\rangle + |\text{alive}\rangle|\text{photon not shot}\rangle)$

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Act 1: (Strong) interaction with a classical system leads to stochastic projective measurement

Measurement operator

Hermitian matrix $Q = \sum_k \lambda_k P_k$

λ_k eigenvalues, P_k projector on eigenspace

Measurement result

output	$y = \lambda_k$	with probability $p_k(Q) = \lvert\langle\psi\lvert P_k\rvert\psi\rangle\rvert$
state	$\lvert\psi\rangle \to P_k\lvert\psi\rangle \,/\, \lVert P_k\lvert\psi\rangle\rVert$	

matrices do not commute $\Rightarrow$ Heisenberg uncertainty principle

$Q_{position}$ eigenvectors $=$ Fourier transform of $Q_{momentum}$ eigenvectors

Act 2: Mixed interaction yields "weak measurement" and control

$$\boxed{\begin{matrix}|\psi(t)\rangle \\ \mathbb{C}^N\end{matrix}} \otimes \boxed{\begin{matrix}|0\rangle \\ \mathbb{C}^2\end{matrix}}$$

state $\otimes$ "actuator"

$\downarrow$ $H_{\text{interaction}}$ on $\mathbb{C}^{2N}$

$$\boxed{\alpha_{0(\psi)}|\xi_{0(\psi)}\rangle \otimes |0\rangle \;+\; \alpha_{1(\psi)}|\xi_{1(\psi)}\rangle \otimes |1\rangle}$$

entangled state

$\downarrow$ $Q_{\text{actuator}} = I \otimes (\lambda_0|0\rangle\langle 0| + \lambda_1|1\rangle\langle 1|)$

$$\boxed{\begin{matrix}|\psi(t+1)\rangle = |\xi_{0(\psi)}\rangle \\ |\psi(t+1)\rangle = |\xi_{1(\psi)}\rangle\end{matrix}}, \quad \boxed{\begin{matrix}y(t) = \lambda_0 \\ y(t) = \lambda_1\end{matrix}} \quad \begin{matrix}\text{with probability } |\alpha_{0(\psi)}|^2 \\ \text{with probability } |\alpha_{1(\psi)}|^2\end{matrix}$$

"weak": $|\xi_{0(\psi)}\rangle$, $|\xi_{1(\psi)}\rangle$ may be just slight modifications of $|\psi(t)\rangle$

Control by interaction allows asymptotic stabilization and feedback

Isolated quantum system

$\psi(T) \;=\; U(T)\,|\psi(0)\rangle$ with $U(t)$ **unitary**

$\Rightarrow$ different $|\psi(0)\rangle$ cannot converge to each other

New possibility with controlled interaction

Measurement operation induces **non-unitary** evolution

$\Rightarrow$ can make different initial conditions converge to target

Measurement result informs on system state $\Rightarrow$ allows feedback.

Act 3: Quantum systems are very fragile to interaction with the environment

Interaction with a “measuring environment” induces system jumps

$t = 0$	$\boxed{\boxed{\lvert\psi_{\text{Syst.}}\rangle} \otimes \lvert\psi_{\text{Env.}}\rangle}$	system $\otimes$ “environment”
	$\downarrow$ $H_{\text{interaction}}$ on $\mathcal{H}_{\text{Syst}} \otimes \mathcal{H}_{\text{Env.}}$	
$t = dt^-$	$\boxed{\sum_k \alpha_k \lvert\xi_{k\text{Syst.}}\rangle \otimes \lvert\xi_{k\text{Env.}}\rangle}$	entangled state
	$\downarrow$ $Q_{\text{Env.}} = I \otimes (\sum_k \lambda_k \lvert k\rangle\langle k\rvert)$	(unknown ??)
$t = dt^+$	$\boxed{\lvert\psi_{\text{Syst.}}\rangle = \lvert\xi_{k\text{Syst.}}\rangle}$ with proba $\lvert\alpha_k\rvert^2$	meas. result $y_{\text{Env.}}$ unknown!!

$\Rightarrow$ jump to unknown state. We can only characterize its “statistical mixture” at dt^+ by $\rho = \sum_k \lvert\alpha_k\rvert^2 \lvert\xi_{k\text{Syst.}}\rangle\langle\xi_{k\text{Syst.}}\rvert$ of rank > 1

The perturbation is particularly bad for states $\lvert\psi_{\text{Syst.}}\rangle$ for which the $\lvert\xi_{k\text{Syst.}}\rangle$ differ a lot.

Unfortunately, this is often the case for the most "natural" states...

The continuous-time limit: coupled stochastic differential equations

Known measurement (feedback control interaction):

$$d\rho = \left(-i[H,\rho] + L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)\right) dt + \left(L\rho + \rho L^\dagger - \mathrm{Tr}\left((L+L^\dagger)\rho\right)\rho\right) dw$$

driven by Wiener process $dw = dy - \mathrm{Tr}\left((L+L^\dagger)\rho\right) dt$
with perfectly correlated sensor output y.

Unknown measurement (disturbance interaction): **Lindblad** master equation

$$\frac{d}{dt}\rho = -\tfrac{i}{\hbar}[H,\rho] + L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)$$

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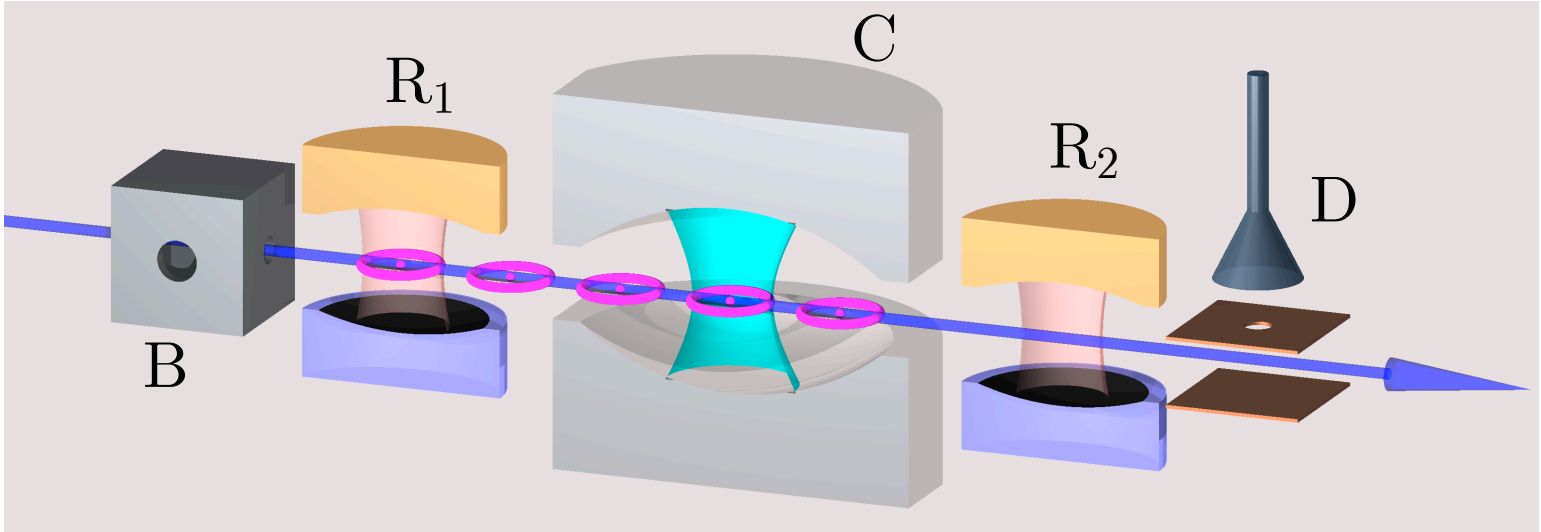
An experiment to control light with atoms[1]

Cavity quantum electrodynamics [CQED] experiment:

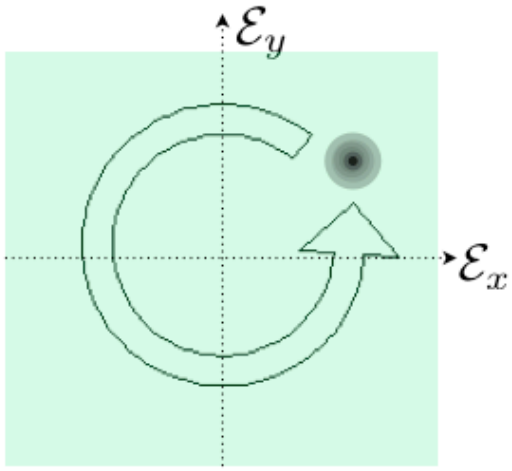
a Microwave Field trapped between "cavity" mirrors is controlled by tailored interaction with Rubidium Atoms sent through the cavity



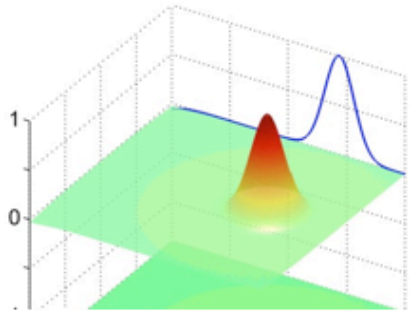
[1] We were fortunate enough to collaborate at Ecole Normale Supérieure, Paris with the "LKB" lab of Serge Haroche, 2012 Nobel laureate in physics.

Goal: stabilize "non-classical" μwave states

Classical electric field: Fresnel diagram with uncertainty ball



Integrating along a radius gives the probability distribution of field amplitude for this quadrature.



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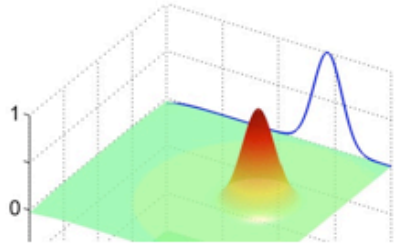
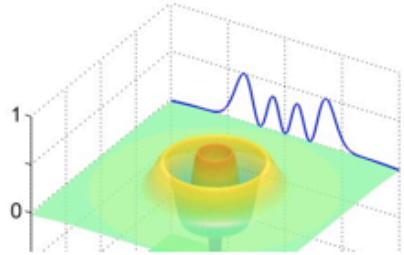
Quantum electric field: Wigner diagram = pseudo-proba-distribution



Integrating along a radius gives the probability distribution of field amplitude for this quadrature.

Different quadratures do not commute, i.e. are not simultaneously observable

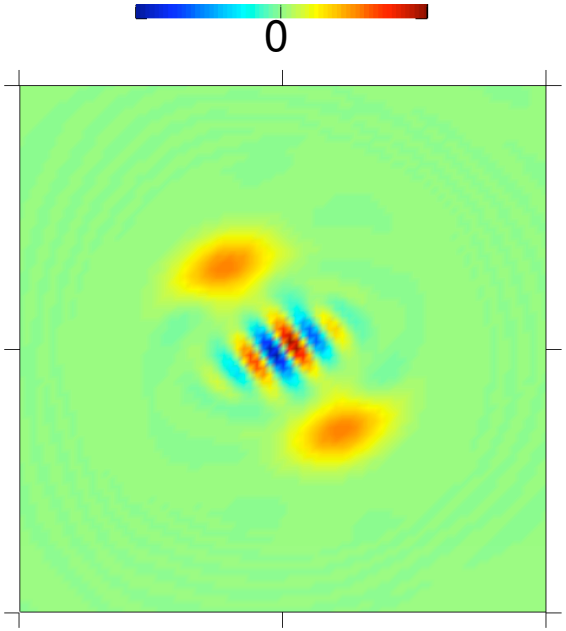


$\Rightarrow$ Wigner function can be negative !!

Goal: stabilize "non-classical" μwave states

A microwave "Schrödinger cat" (SC):

superposition of two opposite amplitudes
with quantum-characteristic negative Wigner pattern in between



get and stay there despite environment-induced "jumps away"

Recall:

$$\boxed{\begin{matrix}|\psi(t)\rangle \\ \mathbb{C}^N\end{matrix}} \otimes \boxed{\begin{matrix}|0\rangle \\ \mathbb{C}^2\end{matrix}}$$

μwave $\otimes$ "actuator" atom

$\downarrow$ $H_{\text{interaction}}$ on $\mathbb{C}^{2N}$: **to be adjusted**

$$\boxed{\alpha_{0(\psi)}|\xi_{0(\psi)}\rangle \otimes |0\rangle \ + \ \alpha_{1(\psi)}|\xi_{1(\psi)}\rangle \otimes |1\rangle}$$

entangled state

$\downarrow$ $Q_{\text{atom}} = I \otimes (\lambda_0|0\rangle\langle 0| + \lambda_1|1\rangle\langle 1|)$

$$\boxed{|\psi(t+1)\rangle = |\xi_{0(\psi)}\rangle}, \quad \boxed{y(t) = \lambda_0} \quad \text{with probability } |\alpha_{0(\psi)}|^2$$

$$\boxed{|\psi(t+1)\rangle = |\xi_{1(\psi)}\rangle}, \quad \boxed{y(t) = \lambda_1} \quad \text{with probability } |\alpha_{1(\psi)}|^2$$

If measurement result not known: $\rho \rightarrow \sum_k |\alpha_k|^2 \, |\xi_{k\text{Syst.}}\rangle\langle\xi_{k\text{Syst.}}|$

Designing a "feedback" controller

... but each (stochastic) observation also (stochastically) moves the system

Two parts in feedback action (a semi-separation principle)

- "Quantum inherent feedback", measuring but ignoring the result: for well-chosen interaction & actuator initial state, the **Kraus** map

$$\rho \rightarrow \sum_k |\alpha_k|^2 \, |\xi_{k\text{Syst.}}\rangle\langle\xi_{k\text{Syst.}}|$$

  can be contracting, asymptotically stabilizing.

  (interpreted like Watt governor: control by interconnection, evacuate 'entropy' of light state through atoms)

- "Active" feedback: adjust actions to measurement results

NB: "coherent feedback" does not make this separation, lets the quantum system interact with a *quantum* controller only

1. Inherent feedback $\sim$ trajectory generation

We iteratively let the system interact with a new atom, tailoring

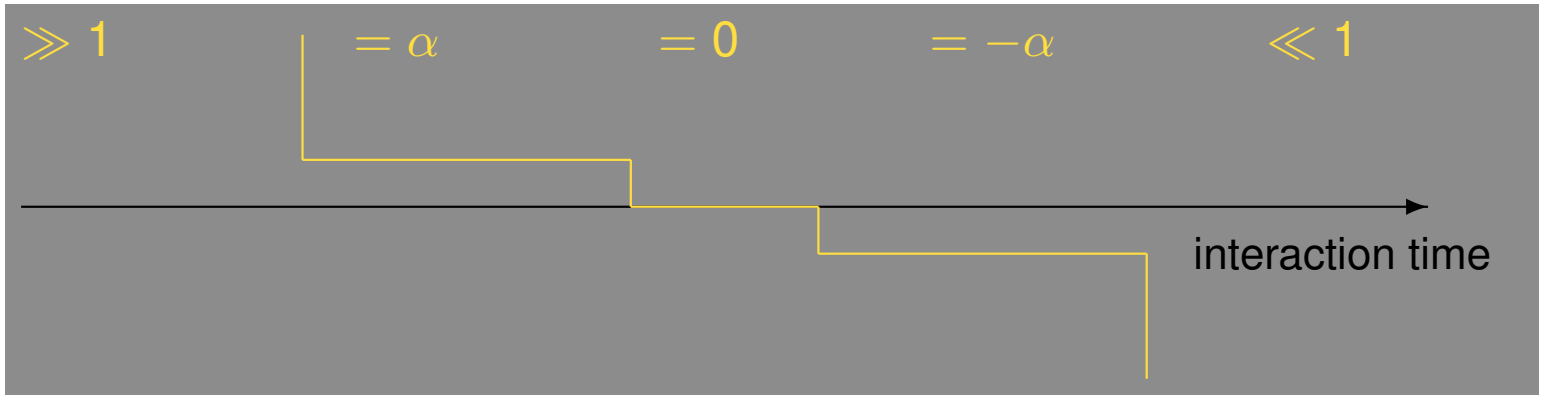
- constant parameter u describing initial atom state

$$|\text{atom}\rangle = \cos(u)|0\rangle + \sin(u)|1\rangle$$

- the interaction between atom and μwave field:

$$H_{int} = \frac{\delta}{2} I \otimes (|1\rangle\langle 1| - |0\rangle\langle 0|) + i\frac{\Omega}{2}(\mathbf{a}^\dagger \otimes |0\rangle\langle 1| + \mathbf{a} \otimes |1\rangle\langle 0|)$$

remarkably, we attain our goal with $\delta(t)$ time-varying as:



Goal: stabilize "non-classical" μwave states

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superposition of two opposite amplitudes
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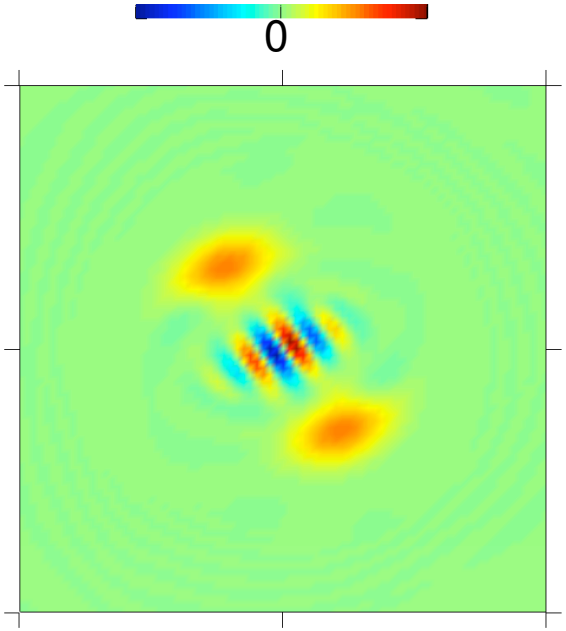


get and stay there despite environment-induced "jumps away"

2. Classical feedback relies on scarce measurement

Limited info by measurement: atom detected in $|0\rangle$ or $|1\rangle$.

To take conclusions from there: know your enemy!

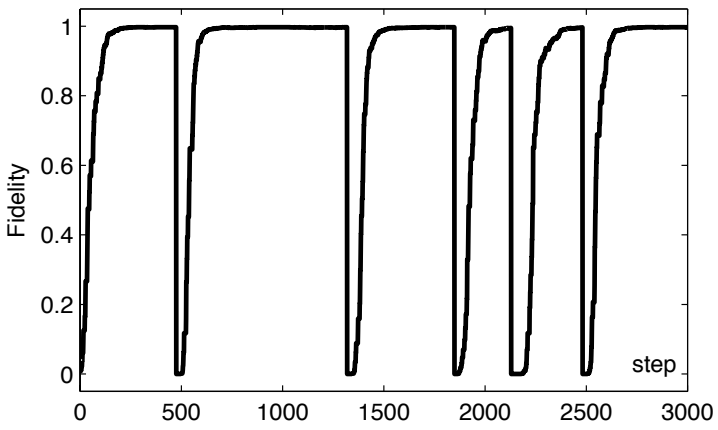
Analysis: most frequent environment-induced jump
is a photon loss
turns the cat by 180°

⇒ Estimation: well-chosen measurement basis Q_{atom} gives
mostly $|0\rangle$ if not turned, mostly $|1\rangle$ if turned

⇒ Action: adjust ref.phase by 180° if we think "something happened"

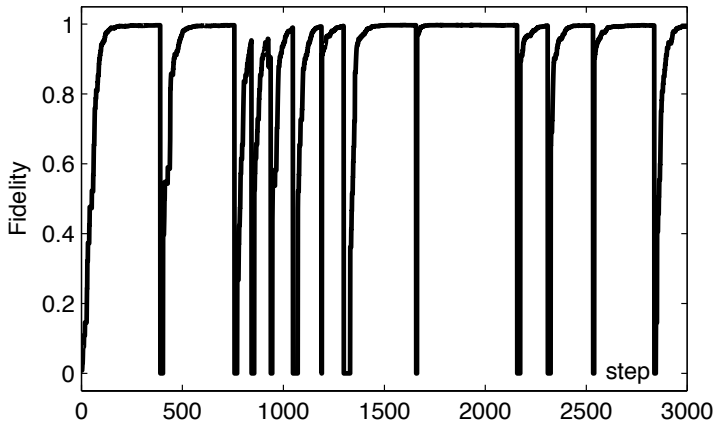
Combining "quantum" & "classical" feedback allows to efficiently stabilize Schrödinger cats in an environment

quantum feedback & env.



average 75 %

+ classical feedback with realistic errors



average 90 %

(simulations)

Recap

1. isolated quantum systems $\sim$ classical model

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3. two types of "feedback", proved experimentally

Challenges for the future

Systematic design methods for quantum control / engineering, especially making robust large networks

Stabilizing not a single state, but a space of states representing (unknown) information: input from / output for interconnected subsystems

Optimal disturbance rejection performance

by using physically available actuation interactions

References:

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Quantum information theory:
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Advanced topics on measurement and feedback:
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Our application:

A. Sarlette, Z. Leghtas, M. Brune, J.M. Raimond and P. Rouchon, *Stabilization of nonclassical states of one- and two-mode radiation fields by reservoir engineering*, Phys.Rev.A vol. 86, pp. 012114 (2012)