RLC: Connections with AC, AL, and RD

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Overview

This seminar aims at highlighting new connections - in terms of stability analysis and convergence proof - underlying the design of the Repetitive Learning Control with respect to well established techniques such Adaptive Control, Adaptive Learning and Regulator Control design.

The problem of unstructured uncertainties

Asymptotic tracking for nonlinear systems might be guaranteed when unstructured (time-invariant) uncertainties appear.

The price to be paid is, however, the adoption of learning control strategies that restrict the output reference signals to be periodic with known periodicity.

In this context, the key idea relies on the use of functional approximations to reproduce the uncertain periodic input reference (*adaptive learning*) or on the voluntary introduction of delays in the control action to improve performance trial by trial (*repetitive learning*).

RLC (1)

Let us start from the output tracking problem in which the output y of the nonlinear time-invariant single input-single output system ($f(\cdot)$ and $g(\cdot)$ are suitable uncertain smooth vector fields on $\mathbb{R}^n$, while $h(\cdot) : \mathbb{R}^n \to \mathbb{R}$ is a suitable uncertain smooth function)

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

is required to track a smooth periodic reference signal $y_*(t)$ (with known period T):

$$y_*(t+T) = y_*(t), \quad \forall\, t \geq -T. \tag{2}$$

In particular, we assume that the global relative degree $\rho \leq n$ is known and well defined for (1) and that system (1) is globally input-output linearizable so that we may directly consider its normal form

$$\begin{aligned} \dot{z} &= \phi(z, \xi) \\ \dot{\xi}_j &= \xi_{j+1}, \quad j = 1, \ldots, \rho - 1 \\ \dot{\xi}_\rho &= L_f^\rho h(x) + L_g L_f^{\rho-1} h(x) u \doteq q(z, \xi) + b(z, \xi) u \\ y &= \xi_1 \end{aligned} \tag{3}$$

in which $z = [z_1, \cdots, z_{n-\rho}]^\top$, $\xi = [\xi_1, \cdots, \xi_\rho]^\top$ are the new vector coordinates, with ξ being available for feedback.

The inverse system dynamics is assumed to constitute a globally exponentially convergent system with the uniformly bounded steady-state property for the class of inputs $\bar{PC}(\cdot)$. For linear inverse system dynamics $\dot{z} = \Gamma z + \Theta\xi$, condition i) is always satisfied when Γ is Hurwitz.

For such a class of systems, the straightforward generalization of the PID$^{\rho-1}$ control ($c(\rho) = 0$ if $\rho = 1$, $c(\rho) = 1$ if $\rho > 1$):

$$\begin{aligned} u(t) &= -k_\rho \left[c(\rho) \sum_{i=1}^{\rho-1} k_i \tilde{y}^{(i-1)}(t) + \tilde{y}^{(\rho-1)}(t) \right] + \hat{u}_*(t) \\ \hat{u}_*(t) &= \mathsf{sat}_{M_u} \left(\hat{u}_*(t-T) \right) \\ &\quad -\mu\varphi(t) \left[c(\rho) \sum_{i=1}^{\rho-1} k_i \tilde{y}^{(i-1)}(t) + \tilde{y}^{(\rho-1)}(t) \right] \\ \hat{u}_*(t) &= 0, \quad \forall\, t \leq 0 \end{aligned} \tag{4}$$

solves (for any initial condition with a properly dependent choice of the constant user-defined control gains k_i, $1 \leq i \leq \rho$) the output tracking problem.

RLC (2)

Now consider the system:

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} x + \chi(y) + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} v \\
&\doteq A_c x + \chi(y) + B_c v \qquad (5) \\
y &= [1, 0, \cdots, 0]x + dv \doteq C_c x + dv
\end{aligned}$$

is considered, where: $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}$ is the output to be controlled, $v \in \mathbb{R}$ is the control input; $\chi(\cdot)$ is an unknown globally Lipschitz smooth vector-valued function with Lipschitz constant L_χ; d is the positive high frequency gain when $\rho = 0$, whereas b_1 is the positive high frequency gain when $\rho = 1$; b_1, $\ldots$, $b_n \in \mathbb{R}^+$ are unknown positive reals such that the n roots of the polynomial

$$p(s) \;=\; ds^n + b_1 s^{n-1} + \ldots + b_n$$

all belong to $\mathbb{C}^-$. Two cases are considered: i) $d > 0$, i.e., the relative degree ρ is equal to zero; ii) $d = 0$ & $b_1 > 0$, i.e., the relative degree ρ is equal to one.

In order to account in (1) for possible model dimension reductions that aim at neglecting fast nonlinear dynamics for the actuator variable v, we allow v to take the form

$$v = u + \varphi(y),$$

where u becomes the input to be designed and $\varphi(\cdot)$ is assumed to be an unknown (globally Lipschitz) monotonic smooth real-valued function with non-positive derivative over its entire domain $\mathbb{R}$

Theorem 1: *Consider system (1) with $\rho = 0$ or $\rho = 1$. Set $v = u + \varphi(y)$. Let y_* be as in Definition 1. Let u_* denote the existing T-periodic unknown reference input for system (1) that guarantees, for compatible initial conditions, perfect output tracking $y(t) \equiv y_*(t)$, for any $t \geq -T$. Let: μ, k_y, M_u be positive control parameters, with $M_u \geq |u_*(t)|$ for any $t \in [0, T)$; $\phi_T(\cdot) : \mathbb{R}^+ \cup \{0\} \to [0, 1]$ be a continuous increasing function for $t \in [0, T]$ such that $\phi_T(0) = 0$ and $\phi_T(t) = 1$ for any $t \geq T$. Then there exists a positive real k_{y*} such that, for $k_y > k_{y*}$, the control ($t \geq 0$):*

$$\begin{aligned} u(t) &= -k_y \tilde{y}(t) + \hat{u}_*(t) \\ \hat{u}_*(t) &= \mathrm{sat}_{M_u}\left(\hat{u}_*(t-T)\right) - \mu\phi_T(t)\tilde{y}(t) \\ \hat{u}_*(q) &= 0, \quad \forall\, q \in [-T, 0) \end{aligned} \tag{6}$$

i) is a repetitive learning control *when $\rho = 1$; ii) is an* exponential repetitive learning control *when $\rho = 0$, while also guaranteeing the asymptotic input tracking:* $\lim_{t\to+\infty}[u(t) - u_*(t)] = 0$ *exponentially.*

What about AC?

The above control constitutes a straightforward generalization of the Proportional-Integral (PI) control that solves the problem when the output reference signal y_* and the input reference u_* are constant.

This can be viewed by neglecting the saturation action, setting $\mu = k_I T$ and taking the limit for $T \to 0$ (if it exists).

In the same way, the arguments used in the first part of the proof of Theorem 1 show the effectiveness of the PI control (excepting for the additional use of PE-based arguments), once the μ-dependent integral quadratic term in $V(t)$ ($\mu = k_I T$):

$$\frac{1}{\mu}\int_{t-T}^{t} (\mathsf{sat}_{M_u}(\widehat{u}_*(\tau)) - u_*(\tau))^2 \mathrm{d}\tau$$

is replaced by the quadratic term $k_I^{-1}\tilde{u}_*^2$.

Different points of view

The RC approach – first proposed at the end of the 1980s – uses a delay as a universal periodic signal generator to achieve asymptotic regulation of the desired output.

When such a delay is viewed as a part of an estimation scheme that is able to provide the estimate $\widehat{u}_*(t)$ of the unknown periodic input reference $u_*(t)$ (while generalizing the classical integral action for constant disturbance remotion), the RC is named RLC to highlight the fact that the controller actually possesses a learning estimation scheme inside.

On the other hand, RLC differs from the so called ILC since in the ILC, the initial conditions of the system are typically set to take the same value at each repetition, while in the RLC, the initial conditions of the system on each trial are set to be equal to the final conditions of the previous repetition.

Different points of view (2)

RLC differs also from ALC, since the latter does not generally achieve convergence to zero of the output tracking error. The output tracking error, in fact, is only guaranteed to be exponentially attracted into a residual connected compact set (containing the origin) whose diameter: i) is zero just in the case of finite Fourier series expansion for the input reference (and any sufficiently high dynamic order of the controller); ii) can be made arbitrarily small by increasing the number N of the estimated Fourier coefficients in the Fourier series expansion for the input reference.

In particular, by properly setting the control gains, the guaranteed output tracking error may be reduced as the Fourier coefficients number is increased, with $\mathcal{L}_2[0,T]$ and $\mathcal{L}_\infty$ transient performance being additionally achieved during the learning phase owing to the use of projection algorithms.

Different points of view (3)

Indeed, the lack of convergence to zero relies on the fact that the ALC strategy represents the input reference, namely a class $\mathcal{C}^{p_y}$ periodic function $\Theta(t)$ with known period T (p_y is any sufficiently large positive integer, $B_\Theta \geq |\Theta(\cdot)|$), as

$$\Theta(t) \;=\; \sum_{l=0}^{N-1} \varrho_l \varphi_l(t) + \varepsilon(t),$$

where: $\varrho[N] = [\varrho_0, \ldots, \varrho_{N-1}]^\top \in \mathbb{R}^N$ is the vector of the first N Fourier coefficients for $\Theta(t)$ ($N > 1$ is an odd number) satisfying

$$\sum_{l=0}^{N-1} \varrho_l^2 \equiv \|\varrho[N]\|^2 \leq \frac{1}{T}\int_0^T \Theta^2(\tau)\mathrm{d}\tau \leq B_\Theta^2;$$

the involved basis functions are given by ($l = 1, 2, \ldots, 2 \leq p \leq p_y$, $\gamma_l = l2\pi/T$):

$$\varphi_0(t) \;=\; 1, \quad \varphi_{2l}(t) \;=\; \sqrt{2}\cos(\gamma_l t), \quad \varphi_{2l-1}(t) \;=\; \sqrt{2}\sin(\gamma_l t);$$

$|\varepsilon(t)| \leq \varepsilon_N$, with the time-independent truncation error* ε_N decreasing to zero as N increases.

*The Fourier series approximation step can be viewed as the mirrored counterpart of the finite memory implementations in the RLC, based on Padé approximants and piecewise linear approximation theory.

Different points of view (4)

Now, the ALC can be viewed as a regulator reformulation that is robust with respect to the approximation error, under a PE condition always verified due to the orthogonal nature of the basis functions. Each sinusoidal contribution to the regulator of the form $\widehat{w}_1$ satisfying:

$$\begin{aligned}\dot{\widehat{w}}_1 &= \widehat{w}_2 - \beta\tilde{y}\\ \dot{\widehat{w}}_2 &= -\gamma^2\widehat{w}_1\end{aligned}$$

can be rewritten as $\widehat{A}_1 \cos(\gamma t) + \widehat{A}_2 \sin(\gamma t)$ under the dynamical equations:

$$\begin{aligned}\dot{\widehat{A}}_1 &= -\beta\cos(\gamma t)\tilde{y}, & \widehat{A}_1(0) &= \widehat{w}_1(0)\\ \dot{\widehat{A}}_2 &= -\beta\sin(\gamma t)\tilde{y}, & \widehat{A}_2(0) &= \gamma^{-1}\widehat{w}_1(0),\end{aligned}$$

whereas each estimation term of the form $\widehat{w}_1$ satisfying:

$$\begin{aligned}\dot{\widehat{w}}_1 &= \widehat{w}_2\\ \dot{\widehat{w}}_2 &= -\beta\tilde{y} - \gamma^2\widehat{w}_1\end{aligned}$$

can be rewritten as $\widehat{A}_1 \cos(\gamma t) + \widehat{A}_2 \sin(\gamma t)$ under the dynamical equations:

$$\begin{aligned}\dot{\widehat{A}}_1 &= \beta\gamma^{-1}\sin(\gamma t)\tilde{y}, & \widehat{A}_1(0) &= \widehat{w}_1(0)\\ \dot{\widehat{A}}_2 &= -\beta\gamma^{-1}\cos(\gamma t)\tilde{y}, & \widehat{A}_2(0) &= \gamma^{-1}\widehat{w}_1(0).\end{aligned}$$

Exp convergence properties/dynamic structure and ALC (Extended Matching)

Consider the class of single input-single output nonlinear systems

$$
\begin{array}{rcll}
\dot{x}_i & = & x_{i+1}, \qquad 1 \le i \le n-1 & \qquad (\text{for } n \ge 2) \\
\theta \dot{x}_n & = & f(x) + u & \\
\pi \dot{u} & = & q(x,u) + v \;=\; q(\xi) + v & \\
y & = & h(x) &
\end{array}
\tag{7}
$$

in which: $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$; $u \in \mathbb{R}$; $\xi = [x^T, u]^T$, $v \in \mathbb{R}$; $y \in \mathbb{R}$; h is a known smooth function; θ and π - here additionally considered with respect to [Marino, Tomei, Verrelli, 2012] - are uncertain constant parameters [of known sign - positive without loss of generality -]; f and q are uncertain smooth functions.

The problem is the one of designing a state $[(x, u)]$ feedback control v in order:
to track (even without a well-defined global relative degree) a reference signal $y_*(t)$ for the output $y(t)$ which is a smooth periodic function of known period T.

The variable u in may be considered as the control variable for the x-subsystem whose uncertain dynamics (forced by the input v) are taken into account: *systems with uncertain actuator dynamics comply with this interpretation.*

Motivating example: Permanent Magnet Step Motors

$$
\begin{aligned}
\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} &= \omega(t) \\
h(\theta(t))\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} &= -\left[\alpha(\theta(t)) + \beta(\theta(t))\omega(t)\right] + i_q(t) \quad + c(\theta(t))i_d(t) \\
\frac{\mathrm{d}i_q(t)}{\mathrm{d}t} &= -\left[\frac{R}{L_0}i_q(t) + \frac{\omega(t)}{L_0}\eta_q(\theta(t))\right] - N_r i_d(t)\omega(t) + \frac{1}{L_0}u_q(t)
\end{aligned}
$$

$$
\frac{\mathrm{d}i_d(t)}{\mathrm{d}t} = -\left[\frac{R}{L_0}i_d(t) + \frac{\omega(t)}{L_0}\eta_d(\theta(t))\right] + N_r i_q(t)\omega(t) + \frac{1}{L_0}u_d(t)
$$

in which the stator windings self inductance L_0 and the number of rotor teeth N_r are the only known parameters and

$$
\begin{aligned}
h(\theta) &= \frac{J}{i_f N_r}\left[\sum_{j=1}^{n} jL_{mj}\cos[(1-j)N_r\theta]\right]^{-1} \\
\alpha(\theta) &= \frac{h(\theta)}{J}\left[T_L(\theta) + \frac{N_r i_f^2}{2}\sum_{j=4}^{n} jL_{fj}\sin[jN_r\theta]\right] \\
\beta(\theta) &= \frac{Dh(\theta)}{J}, \quad c(\theta) \;=\; \frac{h(\theta) i_f N_r}{J}\sum_{j=2}^{n} jL_{mj}\sin[(1-j)N_r\theta] \\
\eta_d(\theta) &= -i_f N_r \sum_{j=2}^{n} jL_{mj}\sin[(j-1)N_r\theta] \\
\eta_q(\theta) &= i_f N_r \sum_{j=1}^{n} jL_{mj}\cos[(j-1)N_r\theta]
\end{aligned}
$$

are uncertain functions. Tracking for the rotor angle $\theta(t)$ of a smooth periodic reference signal of known period is to be guaranteed (along with regulation of $i_d(t)$ to zero).

Latest result: A single learning estimation scheme

It is enough to design an adaptive learning control that just includes a simple adaptive learning estimation scheme in the upper subsystem (through the u-reference u^r), with the tracking error $\tilde{u} = u - u^r$ being deliberately allowed not to converge to zero.

Such $\tilde{u}$ will in turn converge to a T_*-periodic steady-state solution, whose presence will be compensated by the learning action of the upper-subsystem.

Application to the heart rate regulation (open) problem

The presented idea can be used with local modifications to provide an elegant solution to the heart rate regulation problem for treadmill and cycle-ergometer exercises.

- The nonlinear dynamics of a human heart rate during long-duration treadmill exercises have been found in [T.M. Cheng *et al.*] to be described by the experimentally validated second order time-invariant system

$$\begin{aligned} \dot{x}_1 &= -a_1 x_1 + a_2 x_2 + a_6 u^2 \\ \dot{x}_2 &= -a_3 x_2 + a_4 f_{a_5}(x_1) \end{aligned} \tag{8}$$

  in which: the output x_1 is proportional to the heart rate deviation from the heart rate at rest; x_2 is a lumped variable which takes into account slower local peripheral effects on the heart rate response (represented by metabolism, hormones, body temperature and loss of body fluid); u is the treadmill speed. Here a_i, $i = 1, 2, \ldots, 6$, are positive parameters which depend on the specific individual performing the treadmill exercise, while $f_{a_5}(x_1)$ is a globally Lipschitz nonlinear function of the state variable x_1 (with global Lipschitz constant k_{f,a_5}). In particular, $f_{a_5}(x_1)$ is taken in [T.M. Cheng *et al.*] as equal to the RHS of

$$f_{a_5}(x_1) = x_1 \left(1 + \mathrm{e}^{-(x_1 - a_5)}\right)^{-1} .$$

- Model (2) is valid to even describe the dynamics of a human heart rate during cycle-ergometer exercises (in which the cycling speed is kept constant in spite of a varying work load): M. Paradiso, S. Pietrosanti, S. Scalzi, P. Tomei, and C.M. Verrelli, *Experimental heart rate regulation in cycle-ergometer exercises*, IEEE Transactions on Biomedical Engineering, 60: 135-139, 2013. The modifications in interpreting model (2) are the following: the output x_1 is proportional to the heart rate deviation from the heart rate corresponding to the cycling operation under zero work load; x_2 is a lumped variable which takes into account slower local peripheral effects on the heart rate response occurring when passing from the operation at zero work load to the operation at positive work load; u is the work load.

- The proposed approach complies with a scenario in which a simplified DC motor model (ϑ is the rotor position, ω is the rotor speed, T_L is the load torque, $a, b, c, \bar{a}, \bar{b}$ are suitable constants)

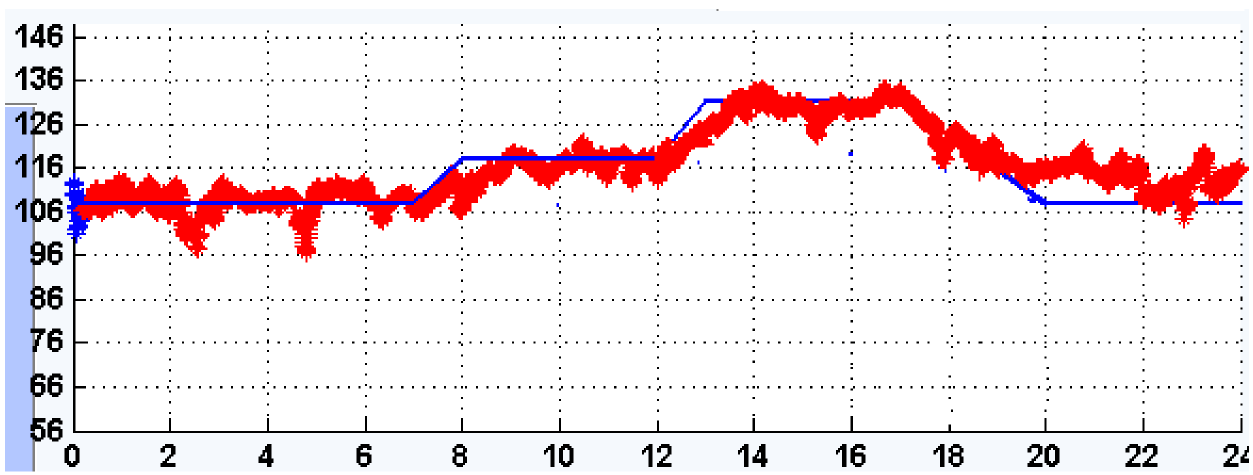
$$\dot{\omega} \quad = \quad av - bT_L + c\omega \tag{9}$$

in the case of negligible motor inductance or

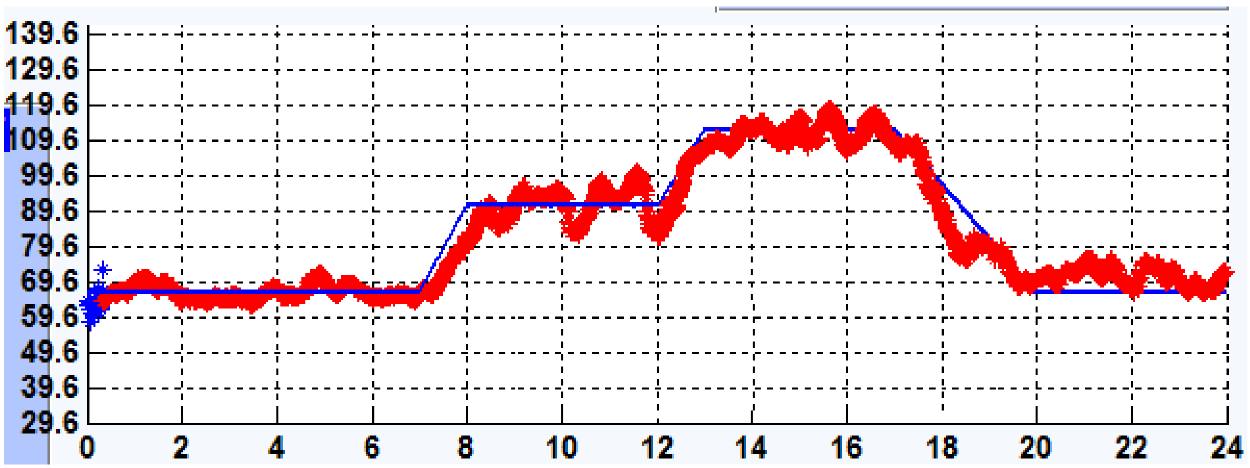
$$\dot{\vartheta} \quad = \quad \bar{a}v - \bar{b}T_L(\vartheta) \tag{10}$$

in the case of negligible motor inductance and inertia constant is also considered. Model (3) concerns the heart regulation problem for treadmill exercises, whereas model (4) concerns the heart regulation problem for cycle-ergometer exercises. Assume, in fact, that there exists suitable functions f_m, g_m, h_m such that $u = f_m(\omega)$ in treadmills or $u = g_m(w)$, $w = h_m(\vartheta)$ in cycle-ergometers (w is the magnet position characterizing the work load).

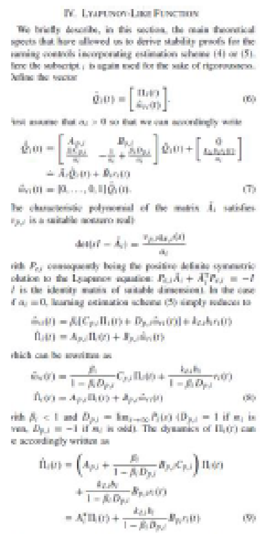
Heart rate and corresponding reference for subject 1.



Heart rate and corresponding reference for subject 2.



Learning control robot control by Pade' approximants


IV. Lyapunov-Like Function

We briefly describe, in this section, the main theoretical spects that have allowed us to derive stability proofs for the earning controls incorporating estimation scheme (4) or (5). Iere the subscript i is again used for the sake of rigorousness. Define the vector

$$\bar{Q}_i(t) = \begin{bmatrix} \Pi_i(t) \\ \hat{w}_{ri}(t) \end{bmatrix}. \quad (6)$$

irst assume that $\alpha_i > 0$ so that we can accordingly write

$$\dot{\bar{Q}}_i(t) = \begin{bmatrix} A_{p,i} & B_{p,i} \\ \frac{\beta_i C_{p,i}}{\alpha_i} & -\frac{1}{\alpha_i} + \frac{\beta_i D_{p,i}}{\alpha_i} \end{bmatrix} \bar{Q}_i(t) + \begin{bmatrix} 0 \\ \frac{k_{\Sigma i} h_i r_i(t)}{\alpha_i} \end{bmatrix}$$
$$= \bar{A}_i \bar{Q}_i(t) + \bar{B}_i r_i(t)$$
$$\hat{w}_{ri}(t) = [0, \ldots, 0, 1]\bar{Q}_i(t). \quad (7)$$

he characteristic polynomial of the matrix $\bar{A}_i$ satisfies $r_{p,i}$ is a suitable nonzero real)

$$\det(sI - \bar{A}_i) = \frac{r_{p,i} d_{m_i}(s)}{\alpha_i}$$

ith $P_{c,i}$ consequently being the positive definite symmetric olution to the Lyapunov equation: $P_{c,i}\bar{A}_i + \bar{A}_i^T P_{c,i} = -I$ I is the identity matrix of suitable dimension). In the case f $\alpha_i = 0$, learning estimation scheme (5) simply reduces to

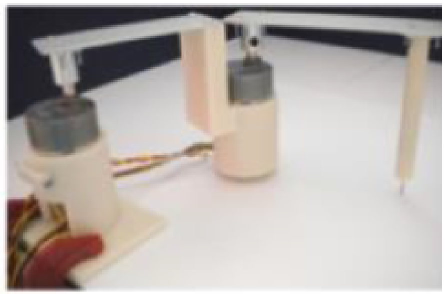
$$\hat{w}_{ri}(t) = \beta_i[C_{p,i}\Pi_i(t) + D_{p,i}\hat{w}_{ri}(t)] + k_{\Sigma i} h_i r_i(t)$$
$$\dot{\Pi}_i(t) = A_{p,i}\Pi_i(t) + B_{p,i}\hat{w}_{ri}(t)$$

hich can be rewritten as

$$\hat{w}_{ri}(t) = \frac{\beta_i}{1 - \beta_i D_{p,i}} C_{p,i}\Pi_i(t) + \frac{k_{\Sigma i} h_i}{1 - \beta_i D_{p,i}} r_i(t)$$
$$\dot{\Pi}_i(t) = A_{p,i}\Pi_i(t) + B_{p,i}\hat{w}_{ri}(t) \quad (8)$$

ith $\beta_i < 1$ and $D_{p,i} = \lim_{s\to\infty} P_i(s)$ ($D_{p,i} = 1$ if m_i is ven, $D_{p,i} = -1$ if m_i is odd). The dynamics of $\Pi_i(t)$ can e accordingly written as

$$\dot{\Pi}_i(t) = \left(A_{p,i} + \frac{\beta_i}{1 - \beta_i D_{p,i}} B_{p,i} C_{p,i}\right)\Pi_i(t) + \frac{k_{\Sigma i} h_i}{1 - \beta_i D_{p,i}} B_{p,i} r_i(t)$$
$$= A_i^* \Pi_i(t) + \frac{k_{\Sigma i} h_i}{1 - \beta_i D_{p,i}} B_{p,i} r_i(t) \quad (9)$$



Picture of the planar manipulator

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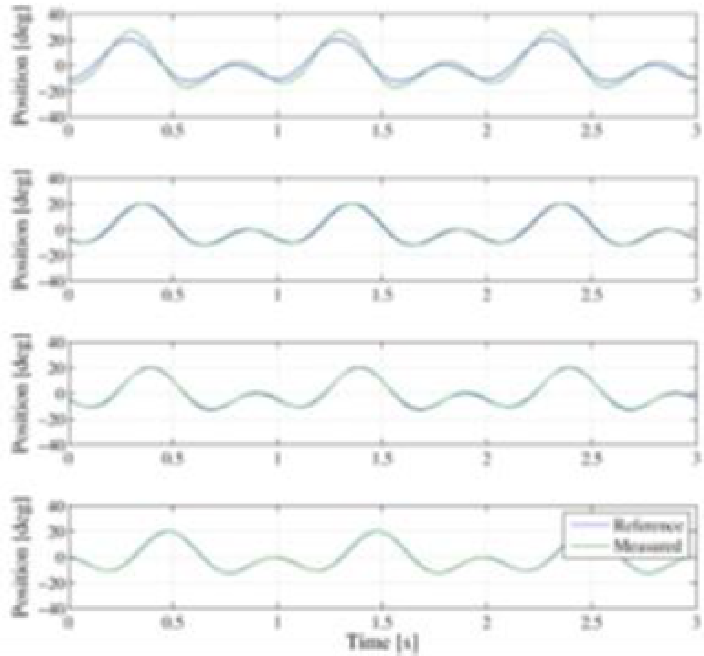
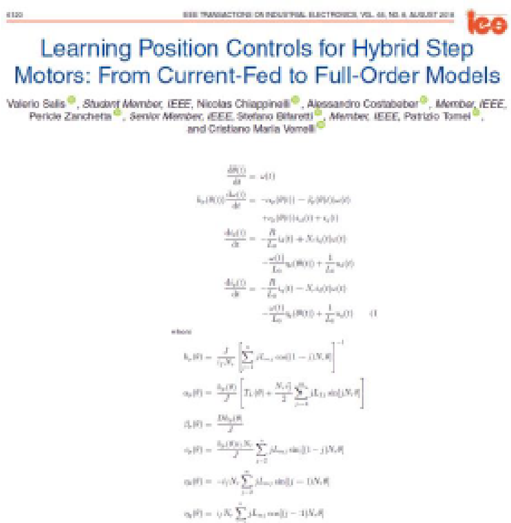
Linear Repetitive Learning Controls for Robotic Manipulators by Padé Approximants

Cristiano Maria Verrelli, Salvatore Pirozzi, Patrizio Tomei, and Ciro Natale

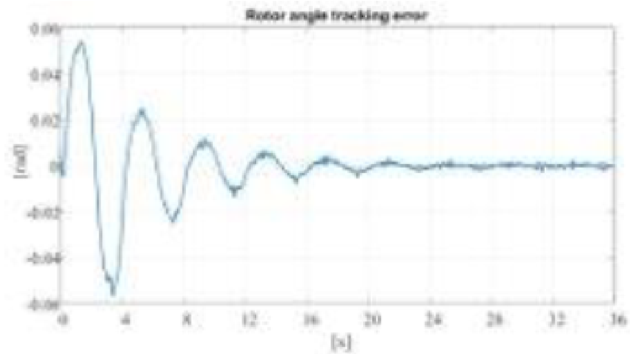


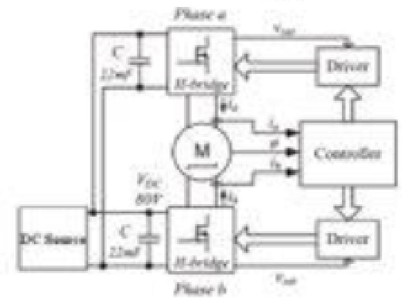
Learning position control for Hybrid Stepper Motors



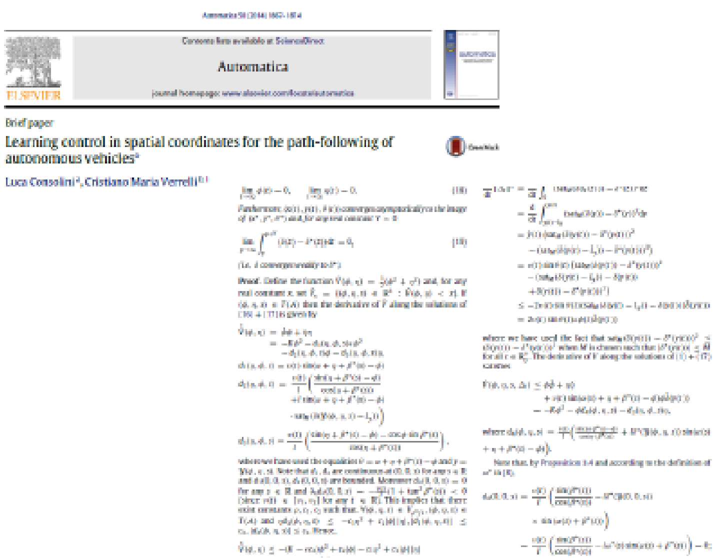
Learning Position Controls for Hybrid Step Motors: From Current-Fed to Full-Order Models

Valerio Salis, Student Member, IEEE, Nicolas Chiappinelli, Alessandro Costabeber, Member, IEEE, Pericle Zanchetta, Senior Member, IEEE, Stefano Bifaretti, Member, IEEE, Patrizio Tomei, and Cristiano Maria Verrelli





Learning Control for Autonomus Vehicles



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Brief paper

Learning control in spatial coordinates for the path-following of autonomous vehicles

Luca Consolini, Cristiano Maria Verrelli