Strongly Stabilizing Controller Design for Distributed Parameter Systems

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Séminaire d'Automatique du Plateau de Saclay

Outline

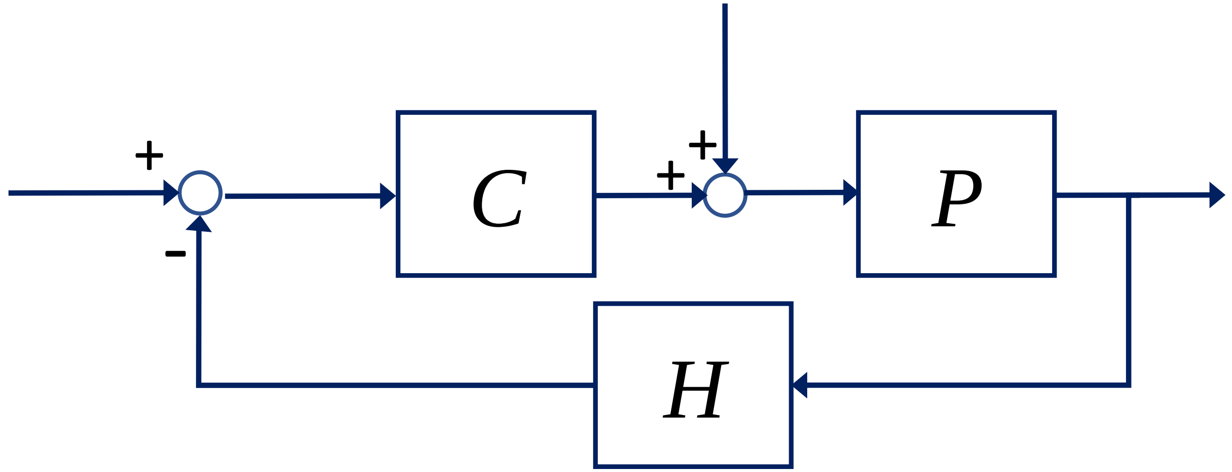
Part I

- What is strong stabilization?
- Unstable plants are “difficult” to control
- Why strong stabilization?
- Strong stabilization methods

Part II

- Distributed parameter systems (DPS) with emphasis on time delays
- Strong stabilization for DPS
- Open research problems

Strong Stabilization



We consider LTI SISO systems, most of the arguments and results apply to LTI MIMO systems as well.

Given *P* and *H*, find a *stable C* stabilizing the feedback system. Stable: transfer function in H_∞ (bounded and analytic in RHP).

Note: if *P* and *H* are stable, *C*=0 stabilizes the feedback system; but this not interesting.

Unstable Plants are “Difficult” to Control

Respect the Unstable

The practical, physical (and sometimes dangerous) consequences of control must be respected, and the underlying principles must be clearly and well taught.

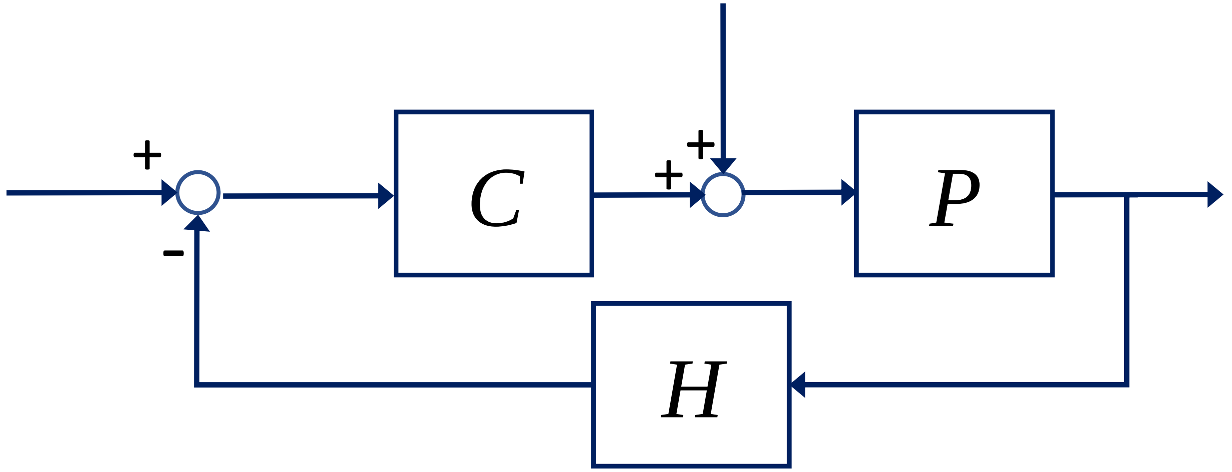
By Gunter Stein

Bode Lecture at the IEEE Conference on Decision and Control in Tampa, Florida, December 1989

The lecture is like really good wine; it ages superbly.” **—Karl J Åström**

IEEE Control Systems Magazine, August 2003, pp. 13—25.

http://ieeecss.org/presentation/bode-lecture/respect-unstable



$$L = CPH \qquad S = (1+L)^{-1}$$

Basic Facts About Unstable Plants

- Unstable systems are fundamentally, and quantifiably, more difficult to control than stable ones.
- Controllers for unstable systems are operationally critical.
- Closed-loop systems with unstable components are only locally stable.

Stein's Bode lecture

Bode Integrals

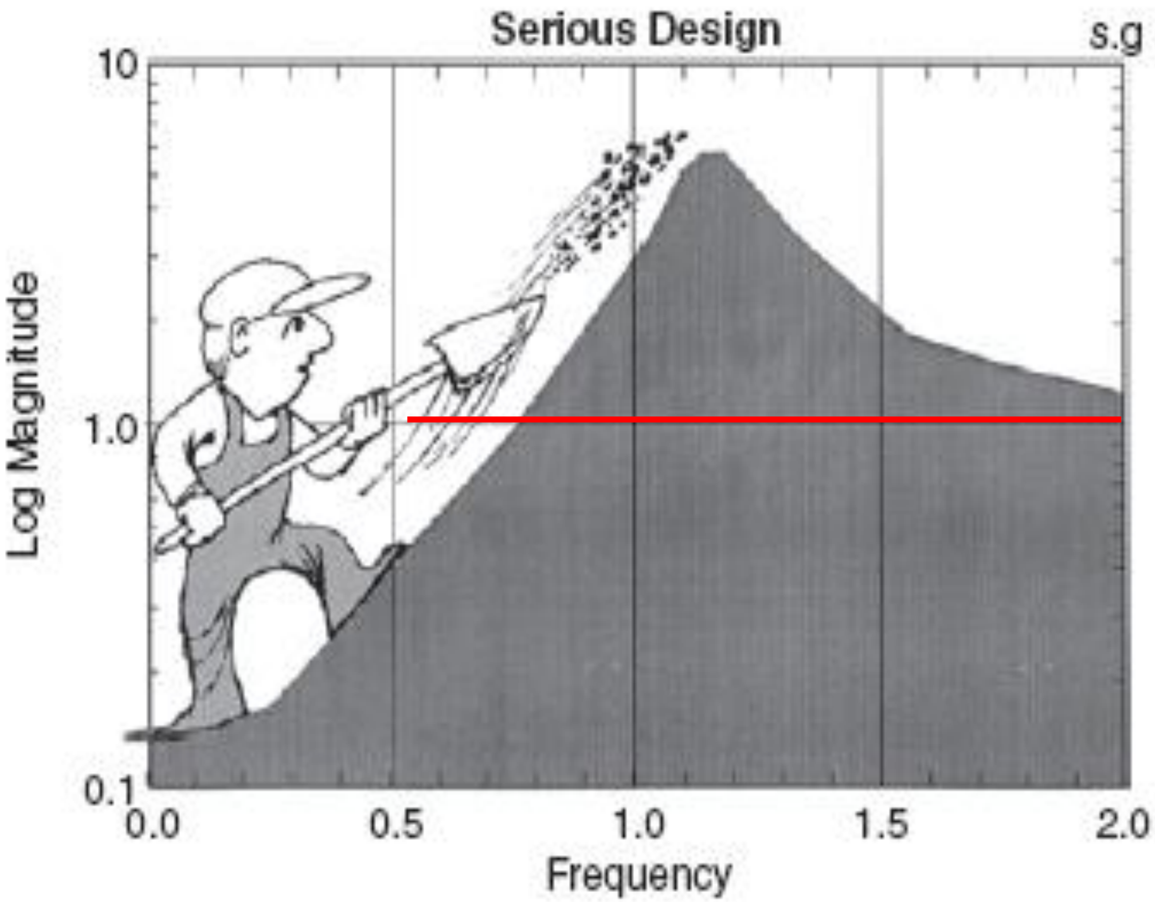
$$\int_0^\infty \ln|S(j\omega)|d\omega = 0 \qquad \text{stable } L = CPH$$

$$\int_0^\infty \ln|S(j\omega)|d\omega = \pi \sum \Re(p_i) \qquad \text{unstable } L \text{ with poles } p_i$$

Meaning of Bode Integrals:

Sensitivity cannot be made less than 1 at all frequencies;

the dirt has to be distributed!



When C is unstable we have to put more dirt above the red line. How much more? It is determined by the real part of the right half plane poles of C.

Use stable controller if possible.

Simple, yet important, examples of unstable plants

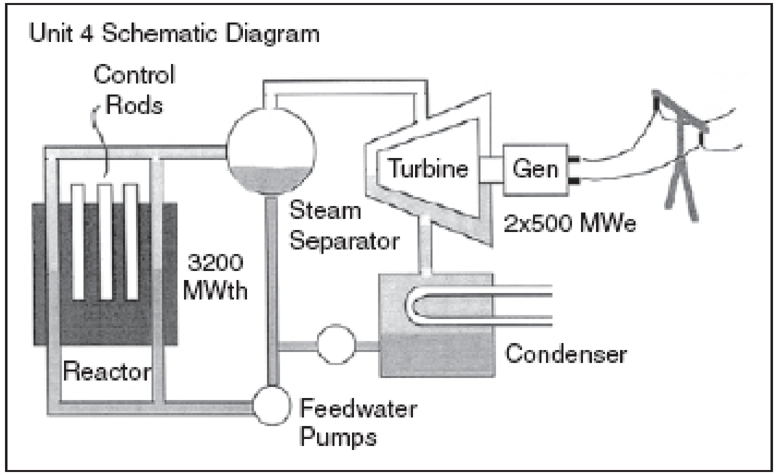


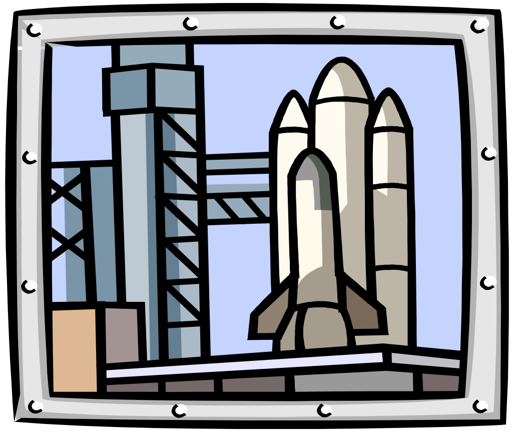
Figure 14. *Schematic diagram of Chernobyl Unit 4 reactor.*

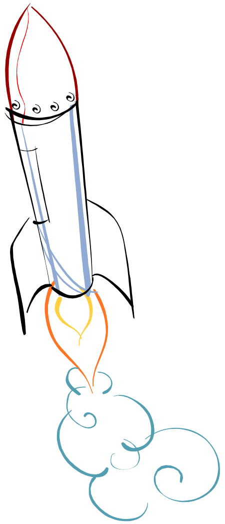
Figure 6. *NASA X-29 forward-swept-wing aircraft (photo courtesy of NASA).*

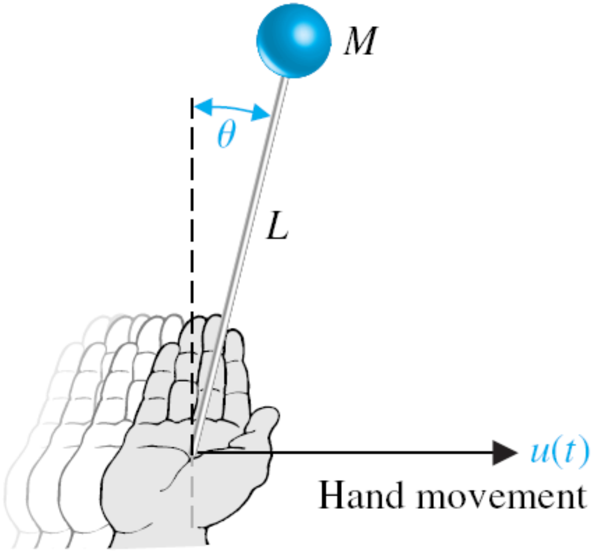
$$P(s)H(s) = \frac{Ke^{-hs}}{(s-a)} \qquad K > 0, \quad a > 0, \quad h > 0$$

$$P(s)H(s) = \frac{Ke^{-hs}}{(s-a)(s+a)}$$

$a = \sqrt{g/L},\qquad h:$ feedback delay

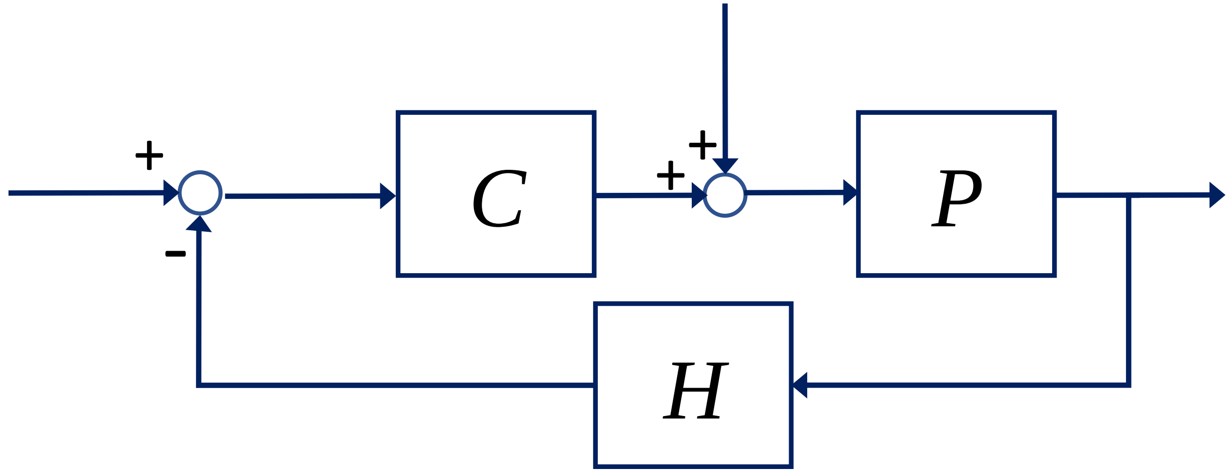






Acrobot and pendubot are higher order unstable extensions.

Important Fact: Aggressive design leads to unstable controllers



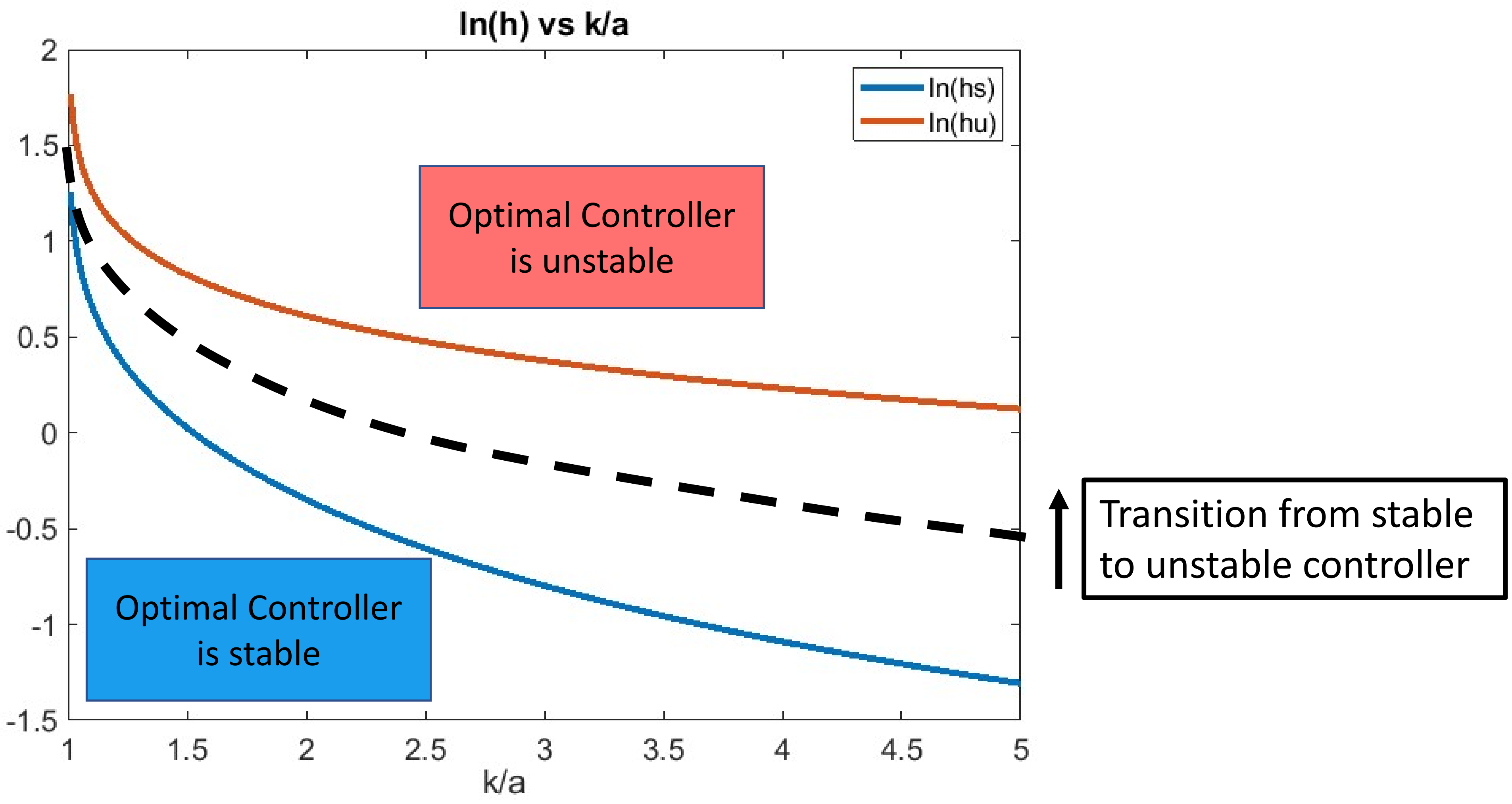
Example: let $H(s)=1$ and consider

$$\gamma_{\text{opt}} = \inf_{(C,P)\ \text{is stable}} \left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty \qquad S = \frac{1}{1+PC} \quad T = 1 - S \qquad P(s) = \frac{e^{-hs/a}}{s+1}$$

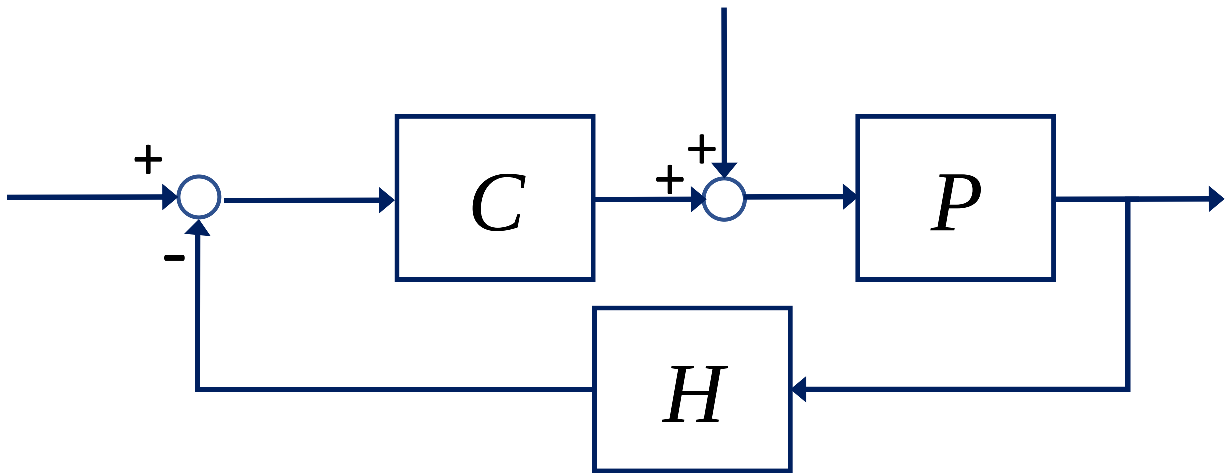
$$W_1(s) = \frac{1 + s/\sqrt{a^2+k^2}}{1+s/a} \sqrt{1+k^2/a^2}$$

$$W_2(s) = (1 + s/\sqrt{a^2+k^2}) \sqrt{1+a^2/k^2}$$

k/a is “large” means aggressive design (more emphasis on the performance, less emphasis on stability robustness)

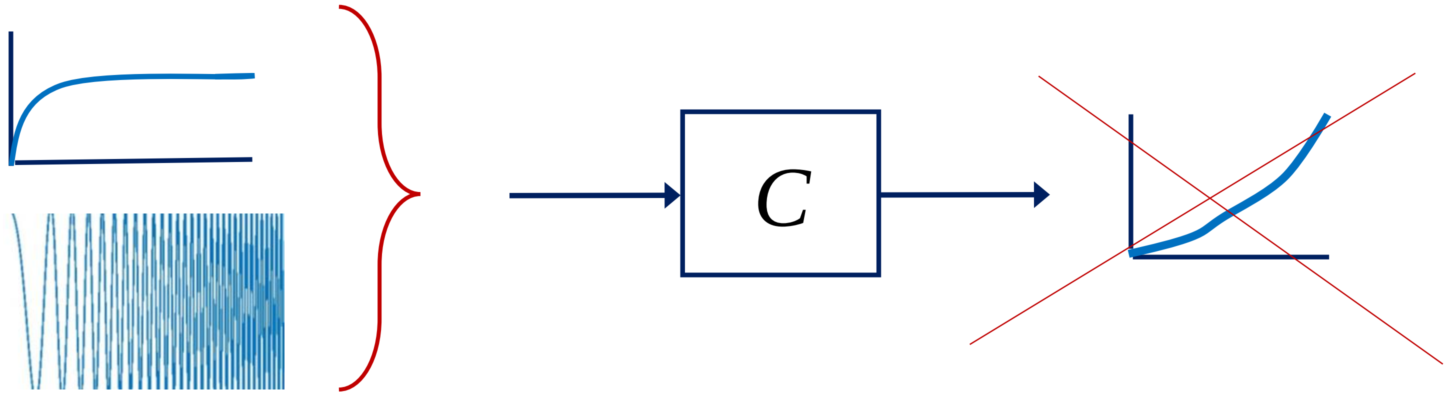

ln(h) vs k/a
ln(hs)
ln(hu)
Optimal Controller
is unstable
Optimal Controller
is stable
Transition from stable
to unstable controller
k/a
2
1.5
1
0.5
0
-0.5
-1
-1.5
1
1.5
2
2.5
3
3.5
4
4.5
5

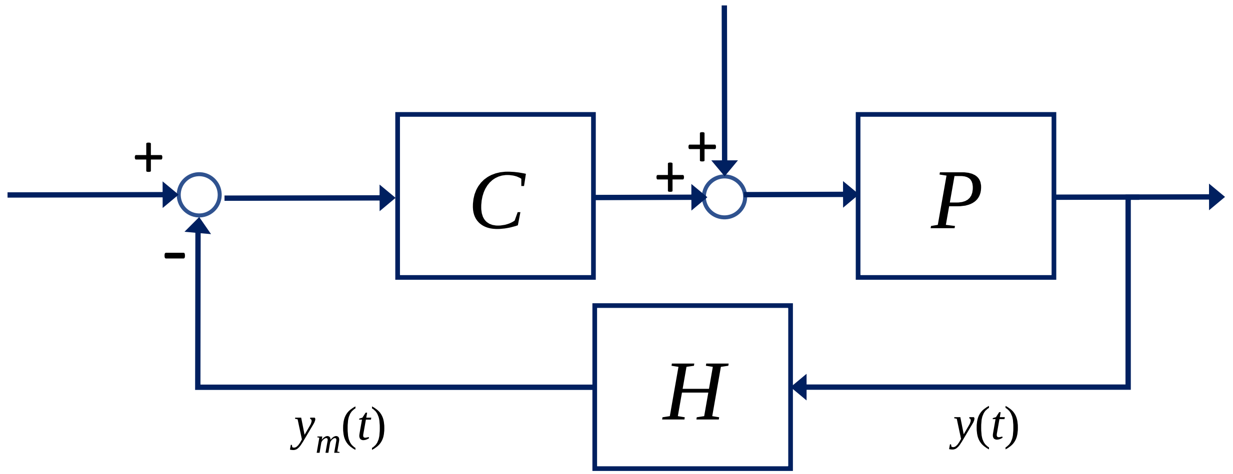
Why Strong Stabilization?

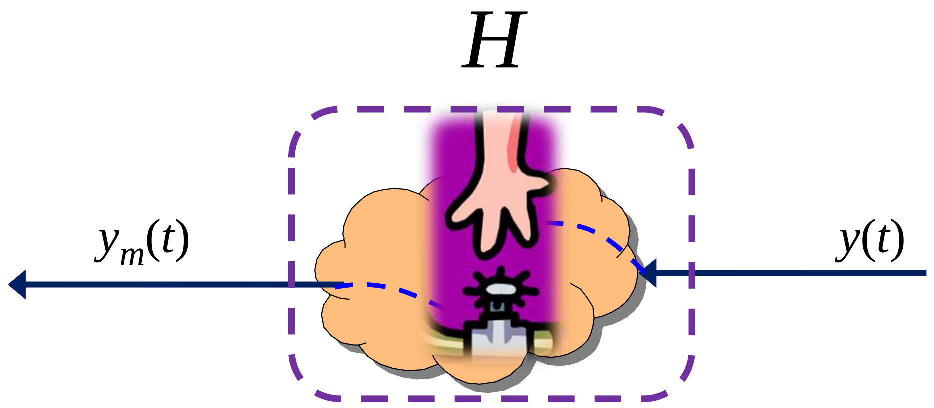


1. Do not introduce extra RHP poles in $L(s)\ldots$Bode integral constraints

2. Off-line testing of the open-loop controller implementation







3. When the plant is stable:
   Preserve stability under sensor failures or malicious attacks

4. Simultaneous stabilization of two plants P_1 and P_2 is equivalent to strong stabilization of an auxiliary plant P_0

5. It is relatively easy to approximate a stable controller and analyze stability robustness of the feedback system

Strong Stabilization Methods

Youla, Bongiorno and Lu (1974):

Plant and sensor: $P(s)H(s)$

There exists a strongly stabilizing controller $C(s)$ for this system

if and only if

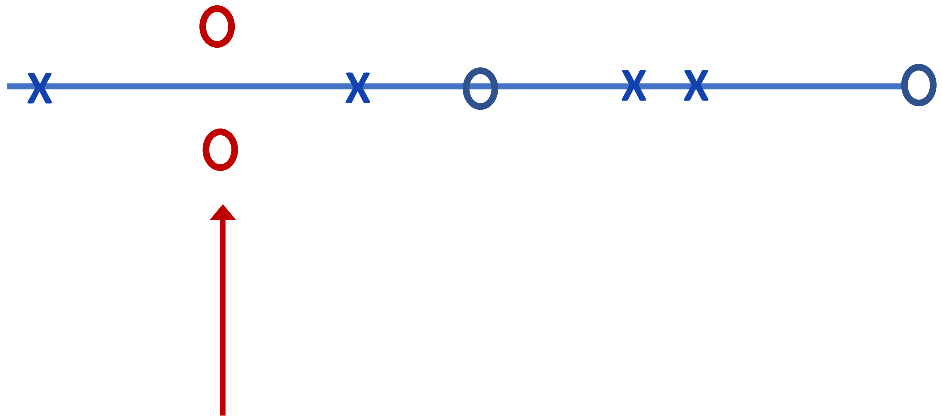
$P(s)H(s)$ satisfies the parity interlacing property (p.i.p), i.e. between every pair of RHP zero there is even number of pole.

PIP is satisfied:

PIP is not satisfied:

Smith and Sondergeld (1986):

PIP is satisfied:



Close to violating the PIP:

As the Im-parts of these zeros get smaller, it becomes *more difficult* to find a strongly stabilizing controller: minimum order of the strongly stabilizing controller increases.

Strongly stabilizing controller design is equivalent to →

$\boxed{\text{Construction of a unit in } H_\infty \text{ satisfying certain interpolation conditions.}}$

Let $P(s)H(s) = N(s)/D(s)$ where N, D are coprime in H_∞; for a given controller $C(s) = Q(s)$ in H_∞ the feedback system is stable if and only if

U, U^{-1} are in H_∞ and $U(z) = D(z)$
for all RHP zeros (including infinity) z of $N(s)$,
where

$$U(s) = D(s) + N(s)Q(s)$$

Sensitivity: $S(s) = D(s)/U(s)$

Controller: $C(s) = (U(s) - D(s))/N(s)$

A simple algorithm for interpolating unit construction
Vidyasagar (1985); see also Doyle-Francis-Tannenbaum (1992)

$N(s)$: $\{ z_1, \dots, z_n \}$ zeros in extended RHP with $z_n = \infty$, *assume they are distinct*

If they are not distinct then we have to deal with higher order interpolation conditions

Intepolation values $\{ D(z_1) =: r_1 , \dots , D(z_n) =: r_n \}$ for simplicity *assume* z_i *are real*.

If they are not real then we have to consider interpolations in complex conjugate pairs.

Assume w.l.o.g. all $r_i > 0$ (p.i.p. holds $\Longleftrightarrow$ they are of the same sign).

Start with $U_1(s) = r_1$

clearly U_1 is unit in H_∞ and satisfies the first interpolation condition.

Step $k > 1$:

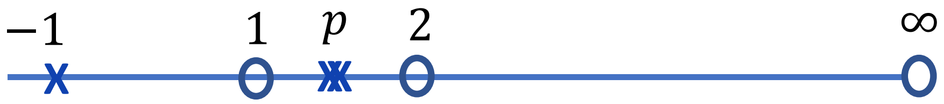
$$U_k(s) = U_{k-1}(s)\,(\,1 + a_k\, F_k(s)\,)^{l_k} \qquad\qquad F_k(s) = \prod_{m=1}^{k-1} \frac{(s - z_m)}{(s + z_m)}$$

Find a_k and l_k with $|a_k| < 1,\; l_k \geq 1,$ such that $U_k(z_k) = r_k$

To satisfy these two conditions we may have to use a large l_k

End with $U_n(s) =: U(s)$

Example



$$P(s)H(s) = \frac{(s-1)(s-2)}{(s-p)^2(s+1)} \qquad N(s) = \frac{(s-1)(s-2)}{(s+1)^3} \qquad D(s) = \frac{(s-p)^2}{(s+1)^2} \qquad \boxed{p = 1.2}$$

$$r_1 = \frac{(1-p)^2}{4}, \qquad r_2 = \frac{(2-p)^2}{9}, \qquad r_3 = 1$$

$$U_1(s) = r_1 = 0.01$$

$$U_2(s) = r_1(\,1 + a_2\ F_2(s)\,)^{l_2} \qquad F_2(s) = \frac{(s-1)}{(s+1)} \qquad U_2(2) = r_2 = 0.07111$$

$a_2 = \underbrace{3}_{1/F_2(z_2)}\,(\,(\underbrace{7.1111}_{\frac{r_2}{r_1}})^{1/l_2} - 1)$ smallest possible l_2 is 8 for $|a_2| < 1 \rightarrow a_2 = 0.833658$

$$U_3(s) = U_2(s)(\,1 + a_3\ F_3(s)\,)^{l_3} \qquad F_3(s) = \frac{(s-1)}{(s+1)}\frac{(s-2)}{(s+2)} \qquad U_3(\infty) = r_3 = 1$$

$$r_3 = U_2(\infty)(\,1 + a_3\ F_3(\infty)\,)^{l_3} \qquad l_3 = 1 \rightarrow a_3 = -\,0.21755$$

$$U(s) = 0.01\left(1 + 0.833658\frac{(s-1)}{(s+1)}\right)^8\left(1 -\ 0.21755\frac{(s-1)}{(s+1)}\ \frac{(s-2)}{(s+2)}\right)$$

10th order

7th order

$$C(s) = (U(s) - D(s))/N(s)$$

$$C(s) = \frac{-1.206\,(s+6.822)\,(s^2\ +\ 1.264s\ +\ 0.4058)\,(s^2 +\ 1.264s\ +\ 0.4818)\,(s^2 +\ 1.297s\ +\ 0.8953)}{(s+2)\,(s+1)^6}$$

$$S = (1 + PC)^{-1} = D\ U^{-1} =$$

$$\frac{(s+1)^{\,7}\,(s+2)(s-1.2)^2}{(s+4.191)\,(s+0.4772)\,(s^2 +\ 0.1441s\ +\ 0.00525)\,(s^2 +\ 0.22s\ +\ 0.01217)(s^2 + 0.1652s\ +\ 0.007171)\,(s^2 +\ 0.1964s\ +\ 0.01001)}$$

Interpolating unit construction using infinite dimensional transfer functions Vidyasagar (1985), see also Ganesh and Pearson (1986)

Choose $U(s) = \exp(F(s))$ such that $|F(s)| \leq M, \forall\ Re(s) \geq 0$, for some $M > 0$ and

$$U(z_i) = r_i = \exp(F(z_i))$$

$$r_i = |r_i| \exp(\, j\, (\varphi(r_i) + 2\,\pi\, m_i)\,) \qquad \frac{-\pi}{2} < \varphi(r_i) < \frac{\pi}{2}$$

$$F(z_i) = ln(r_i) = ln(|r_i|) + j\varphi(r_i) + j\, 2\,\pi\, m_i$$

Lagrange interpolation

Back to the Example

$N(s) = \frac{(s-1)(s-2)}{(s+1)^3}$ $\quad D(s) = \frac{(s-1.2)^2}{(s+1)^2}$ $\quad r_1 = \frac{(1-1.2)^2}{4}, \quad r_2 = \frac{(2-1.2)^2}{9}, \quad r_3 = 1$

$f_1 = ln(r_1)\,, f_2 = ln(r_2)\,, \;\; f_3 = ln(r_3) = 0$ $\Rightarrow$ $F(s) = \frac{as+b}{(s+1)^2}$

$f_1 = \frac{az_1+b}{(z_1+1)^2}\,, f_2 = \frac{az_2+b}{(z_2+1)^2}$ $\Rightarrow$ find a and b $\quad F(s) = \frac{-5.3709\,(s+2.43)}{(s+1)^2}$

Back to the Example

$$N(s) = \frac{(s-1)(s-2)}{(s+1)^3} \qquad D(s) = \frac{(s-1.2)^2}{(s+1)^2} \qquad r_1 = \frac{(1-1.2)^2}{4}, \qquad r_2 = \frac{(2-1.2)^2}{9}, \qquad r_3 = 1$$

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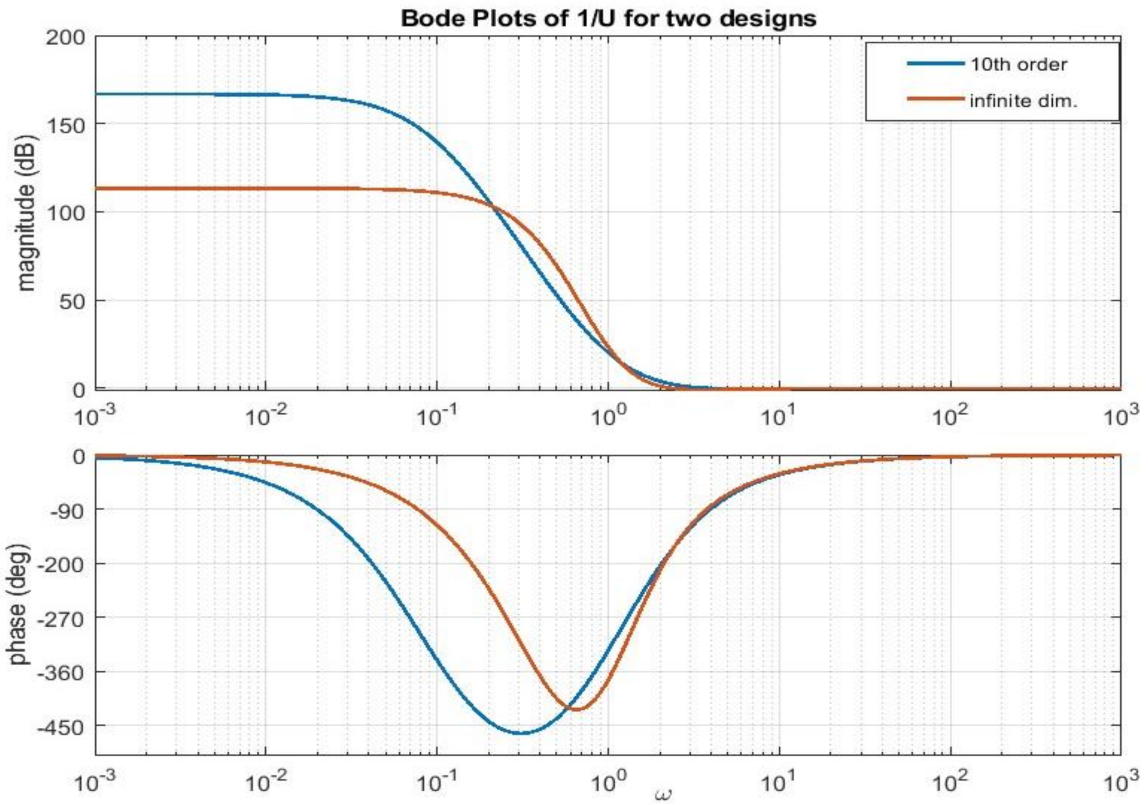
$$f_1 = \frac{az_1+b}{(z_1+1)^2}\,,\, f_2 = \frac{az_2+b}{(z_2+1)^2} \quad \Rightarrow$$ find a and b $$F(s) = \frac{-5.3709\,(s+2.43)}{(s+1)^2}$$

$U(s) = \exp(F(s))$

$U^{-1}(s) = \exp(-F(s))$

U^{-1} is very large at $s = 0$



Can we put a bound on the H_∞ norm of U and U^{-1}?

Interpolating unit construction using the Nevanlinna-Pick method

$U(s)$ must be bounded and analytic on $\mathbb{C}_+$ and must satisfy $U(z_i) = r_i$ and

$$\varepsilon < |U(s)| < \gamma \,, \forall \; Re(s) \geq 0, \text{for some } \gamma > \varepsilon > 0$$

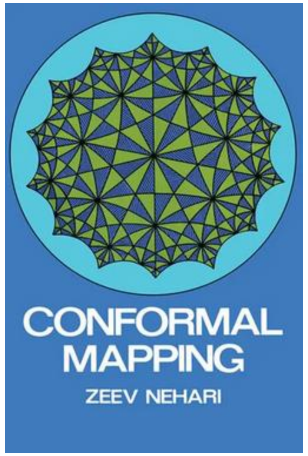
$$U : \overline{\mathbb{C}}_+ \rightarrow \mathbb{W}_\gamma \qquad \mathbb{W}_\gamma := \{re^{j\theta} \in \mathbb{C} : \quad \varepsilon < r < \gamma \,, \quad -\pi < \theta < \pi\}$$

$$\|U\|_\infty < \gamma$$

$$\|U^{-1}\|_\infty < \varepsilon^{-1}$$

$$\begin{tikzcd}
\overline{\mathbb{C}}_+ \arrow[rrr, "U"] \arrow[ddd, "\varphi"'] & & & \mathbb{W}_\gamma \arrow[ddd, "\phi_\gamma"] \\
& z_i & r_i & \\
& \alpha_i & \beta_i & \\
\overline{\mathbb{D}} \arrow[rrr, "\vartheta"'] & & & \mathbb{D}
\end{tikzcd}$$

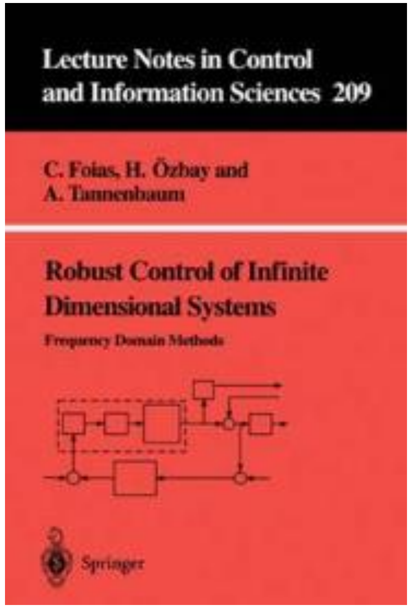
φ and ϕ_γ are conformal maps, for construction see Nehari's book (1952); see also: A. Ringh, J. Karlsson and A. Lindquist, *IEEE T-AC,* Jan. 2022.



Using conformal mappings the problem is reduced to finding

$$\vartheta : \overline{\mathbb{D}} \to \mathbb{D} \qquad \text{such that} \quad \vartheta(\alpha_i) = \beta_i, \quad i = 1, \ldots, n.$$

This is a Nevanlinna-Pick problem, see e.g.



Final solution is given by $\boxed{U(s) = \phi_\gamma^{-1}(\vartheta(\varphi(s)))}$

The result is an infinite dimensional $U(s)$ because ϕ_γ is infinite dimensional.

We approximate it to get a finite dimensional strongly stabilizing controller.

For examples see:

H. Özbay, *LNCIS* vol. 398, pp. 105–113, Springer-Verlag, 2010.

S. Gümüşsoy and H. Özbay, *IEEE T-AC,* vol. 54, March 2009.

Robustness Analysis

Uncertain plant: $P_\Delta = \dfrac{N\ (1+W_2\Delta_N)}{D\ (1+W_1\Delta_D)}$ $\quad W_1, W_2, \Delta_D, \Delta_N \in \mathcal{H}^\infty \quad \|[\Delta_D \quad \Delta_N]\|_\infty < 1$

W_1, W_2: uncertainty weights

U is designed from $P_o = N/D$, and $C = (U-D)/N$, resulting in $S_o = DU^{-1}$, $T_o = 1 - S_o$

Sensitivity function for the system formed by controller C and plant P_Δ

$$S_\Delta = (1+W_1\Delta_D)\ S_o \left(1 + \ [\Delta_D \quad \Delta_N] \begin{bmatrix} W_1 S_o \\ W_2 T_o \end{bmatrix}\right)^{-1}$$

Perturbed feedback system is stable if and only if

$$\left\| \begin{bmatrix} W_1 S_o \\ W_2 T_o \end{bmatrix} \right\|_\infty \leq 1 \quad \Longrightarrow$$

Additional design requirement: Minimize the LHS (mixed sensitivity minimization)

This brings us to the problem of H_∞ control with strongly stabilizing controllers (difficult open problem)

Strong stabilization under sufficient conditions (small gain)

$$P_o = N/D, \text{ s.t. } D \in \mathcal{H}^\infty \text{ with } D(\infty) = 1$$

$$U = (D + NQ) \quad C = Q$$

$$\text{Choose an arbitrary } W_o, W_o^{-1} \in \mathcal{H}^\infty, \text{ s.t. } W_o(\infty) = 1$$

$$W_o U \text{ is unitary in } \mathcal{H}^\infty \Longleftrightarrow U \text{ is unitary in } \mathcal{H}^\infty$$

$$Q \in \mathcal{H}^\infty \Longleftrightarrow Q_1 = W_o Q \in \mathcal{H}^\infty$$

Define $W_o D =: 1 - R$; $\qquad W_o U = W_o D + N W_o Q = 1 - (R - NQ_1)$

Then $Q = W_o^{-1} Q_1$ is a strongly stabilizing controller if $Q_1 \in \mathcal{H}^\infty$ satisfies

$$\|(R - NQ_1)\|_\infty < 1$$

One-block H_∞ control problem: Nehari, Nevanlinna-Pick, Sarason or Commutant Lifting theorems can be used (infinite dimensional $N(s)$ can also be handled).

Extension to two block

$$\|Q\|_\infty < \rho \qquad \Longleftrightarrow \qquad |\rho^{-1} W_o^{-1}(j\omega) Q_1(j\omega)| < 1 \quad \forall\, \omega$$

$$\left\| \begin{bmatrix} R \\ 0 \end{bmatrix} - \begin{bmatrix} N \\ -\rho^{-1} W_o^{-1} \end{bmatrix} Q_1 \right\|_\infty < 1$$

$$\Longrightarrow \qquad \|Q\|_\infty < \rho \qquad \text{and} \qquad \|(R - NQ_1)\|_\infty < 1$$

Two-block H_∞ control problem:
its solution is obtained from a spectral factorization + one-block H_∞ control problem

If the problem is solvable for some ρ_o then it is solvable for all $\rho \geq \rho_o$.

Allows parameterization of a set of strongly stabilizing controllers

MIMO version of the small gain approach in state space (Zeren and Özbay, 2000)

$$\left\| \begin{bmatrix} R \\ 0 \end{bmatrix} - \begin{bmatrix} N \\ -\rho^{-1} W_o^{-1} \end{bmatrix} Q_1 \right\|_\infty < 1$$

Take $W_o = I$, and $P(s) = C(sI - A)^{-1}B$, with (A, B) controllable, (C, A) observable.

$A^{\mathrm{T}}X + XA - XBB^{\mathrm{T}}X = 0$ $\qquad$ $X = X^{\mathrm{T}} \succeq 0$ and $A_X = A - BB^{\mathrm{T}}X$ is stable.

$P = ND^{-1}$, $\quad N(s) = C(sI - A_X)^{-1}B$, $\quad D(s) = F(sI - A_X)^{-1}B + I$, $\quad F = -B^{\mathrm{T}}X$

MIMO version of the small gain approach in state space (Zeren and Özbay, 2000)

$$\left\| \begin{bmatrix} R \\ 0 \end{bmatrix} - \begin{bmatrix} N \\ -\rho^{-1} W_o^{-1} \end{bmatrix} Q_1 \right\|_\infty < 1$$

Take $W_o = I$, and $P(s) = C(sI - A)^{-1}B$, with (A, B) controllable, (C, A) observable.

$A^T X + XA - XBB^T X = 0$ $\qquad X = X^T \succeq 0$ and $A_X = A - BB^T X$ is stable.

$P = ND^{-1}$, $\quad N(s) = C(sI - A_X)^{-1}B$, $\quad D(s) = F(sI - A_X)^{-1}B + I$, $\quad F = -B^T X$

If a stabilizing solution $Y = Y^T \succeq 0$ exists for the ARE

$$A_X Y + Y A_X^T - Y(\rho^2 C^T C - XBB^T X)Y + BB^T = 0$$

for some $\rho > 0$, then a strongly stabilizing controller is $K(s) = F(sI - A_K)^{-1}L$

n^{th} order controller

$$A_K = A + BF + LC \ , \quad L = -\rho^2 Y C^T \ .$$

Closed loop system poles are $\lambda(A_X)$ and $\lambda(A_Y)$, with $A_Y = A - \rho^2 Y C^T C$; moreover $\|K\|_\infty \leq \rho$

If the problem is solvable for some ρ_o then it is solvable for all $\rho \geq \rho_o$.

Allows parameterization of a set of strongly stabilizing controllers

There are many other methods for strongly stabilizing controller design.
For further references and literature review see recently published papers on this topic:

Hakkı Ulaş Ünal,
"On Stable H∞ controller design for plants with infinitely many unstable zeros and poles"
Automatica, vol. 138 (2022), 110036.

Nazlı Gündeş and Hitay Özbay,
"Strong Stabilization of High Order Plants"
Automatica, vol. 140 (2022), 110256.

Jovan D. Stefanovski,
"Interpolation with strongly F-positive real matrix and application to strong stabilization"
Automatica, vol. 154 (2023), 111093.

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- Unstable plants are “difficult” to control
- Why strong stabilization?
- Strong stabilization methods

Part II

- Distributed Parameter Systems (DPS) and Time Delay Systems (TDS)
- Strong stabilization for DPS
- Open research problems

Gunter Stein said

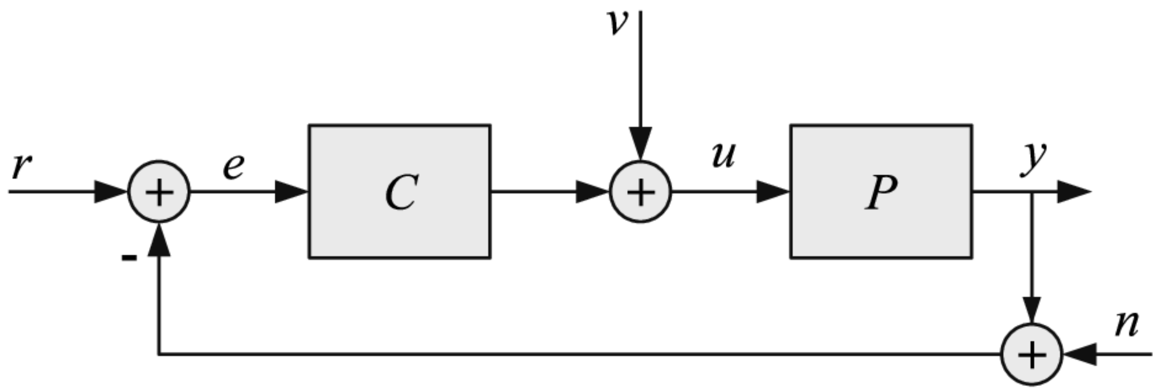
Respect the Unstable

The practical, physical (and sometimes dangerous) consequences of control must be respected, and the underlying principles must be clearly and well taught.

From this perspective we should also

Respect the Delay

The Delay Margin characterizes
a fundamental limitation of the feedback system:
aggressive controller design is dangerous for systems with delay



Delay Margin Optimization (an open problem):

Given a plant $P(s)$ find a controller $C(s)$ such that the feedback system is stable with largest possible delay margin.

DM improvement: H. Özbay, N. Gündeş, *Automatica* (2020)
Lower bound for achievable DM: A. Ringh, J. Karlsson, A. Lindquist, *IEEE T-AC* (Jan-2022)
DM optimization for low order systems: J. Chen, D. Ma, Y. Xu, J. Chen, *IEEE T-AC* (March 2022)

Application Areas of DPS and TDS

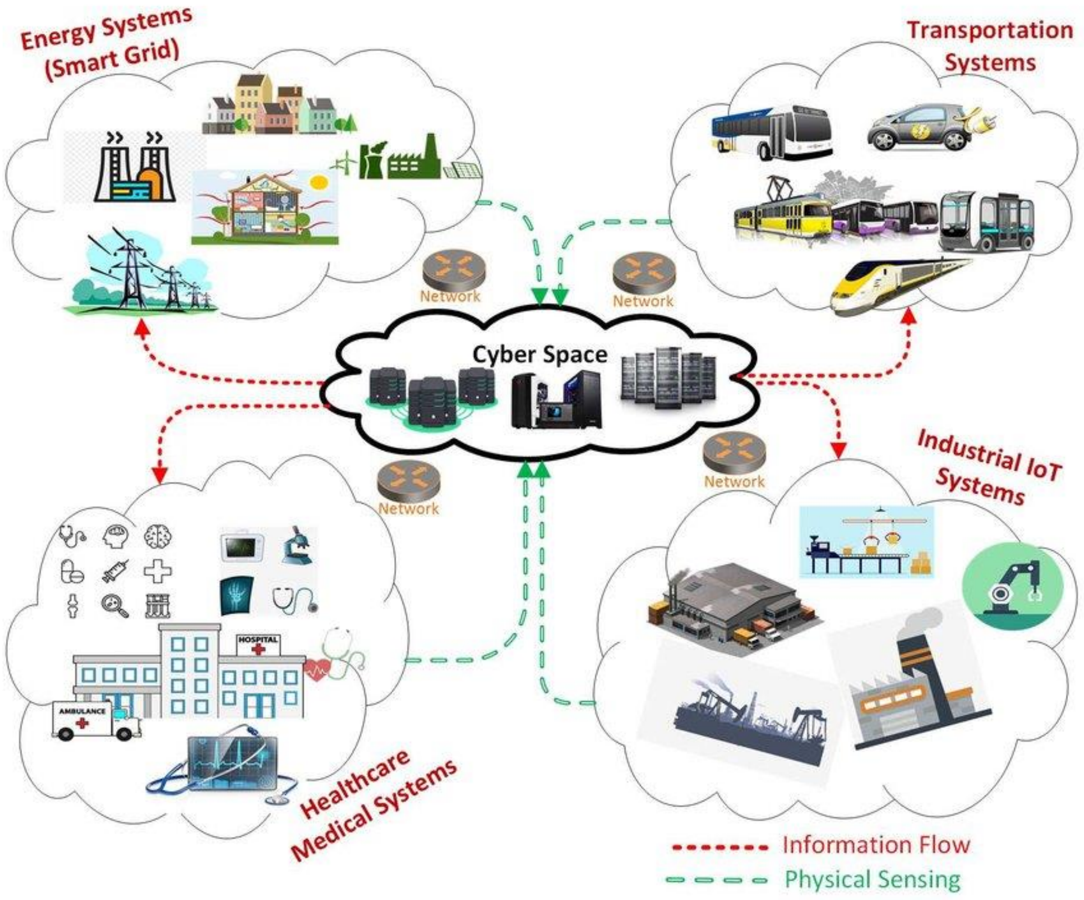
NSF website: **https://www.nsf.gov/news/special_reports/cyber-physical/**

CYBER-PHYSICAL SYSTEMS:
Enabling a Smart and Connected World

Cyber-physical systems integrate sensing, computation, control and networking into physical objects and infrastructure, connecting them to the Internet and to each other.

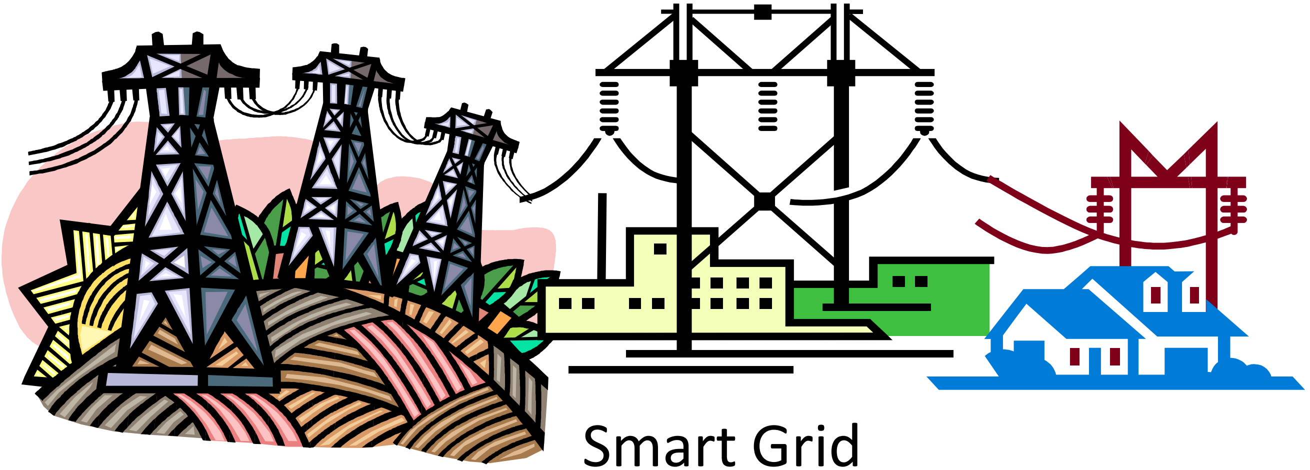
Some application areas:

- Transportation and energy
- Healthcare and medicine
- Environment and sustainability
- Manufacturing

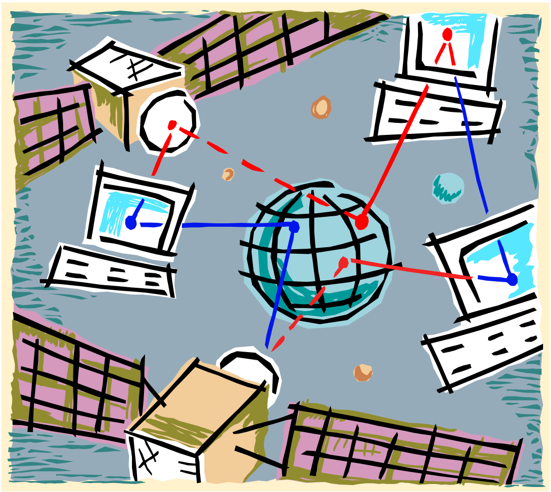


Networked Control Systems

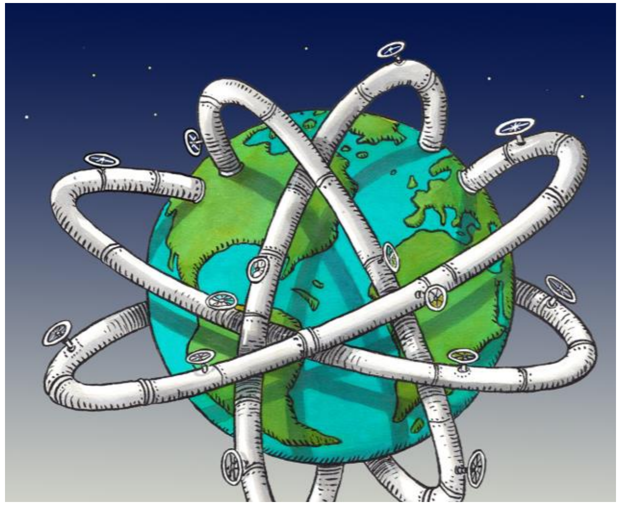
Examples of Networked Control Systems



Smart Grid



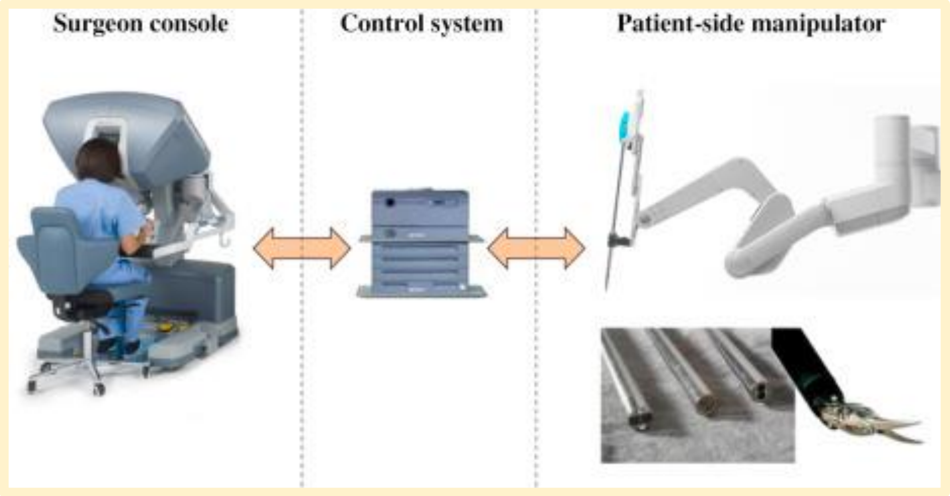
Communication Networks



Gas/Oil Pipelines

Transportation

Examples of Networked Control Systems

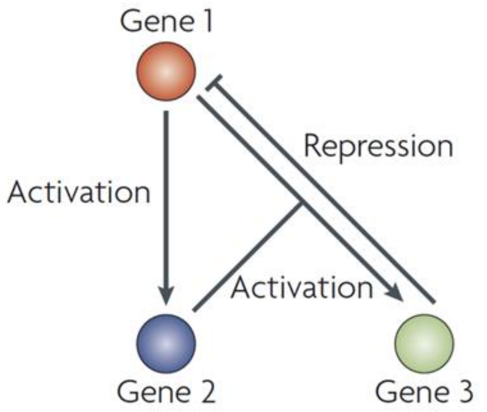


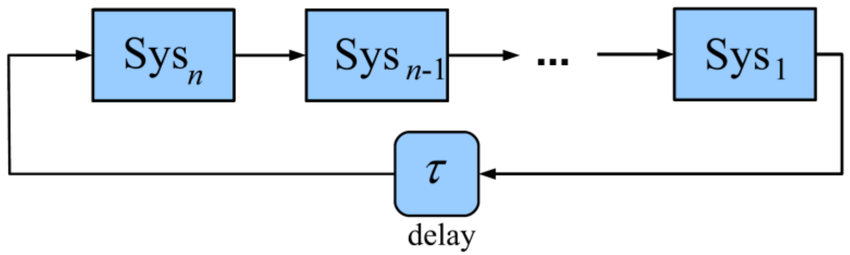
Tele-robotics

and many other applications involving tele-operation

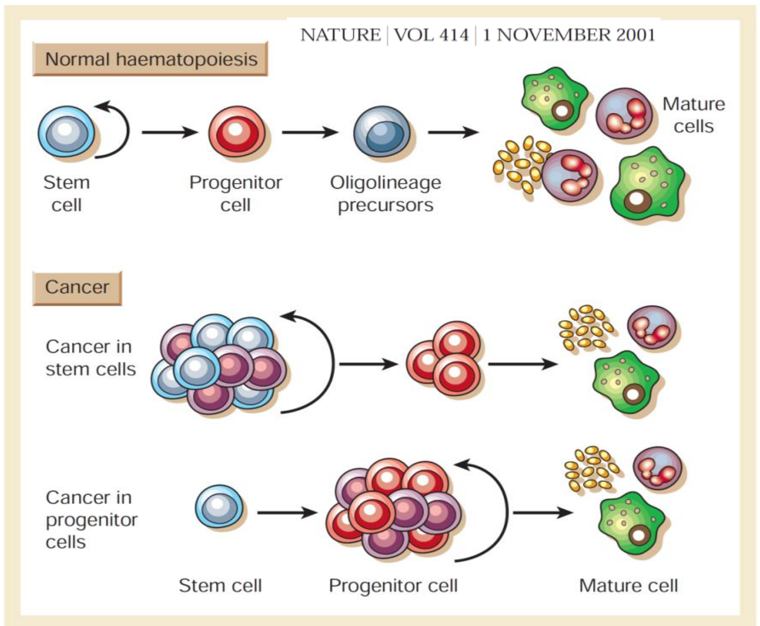
Examples of Biological Systems with Delays

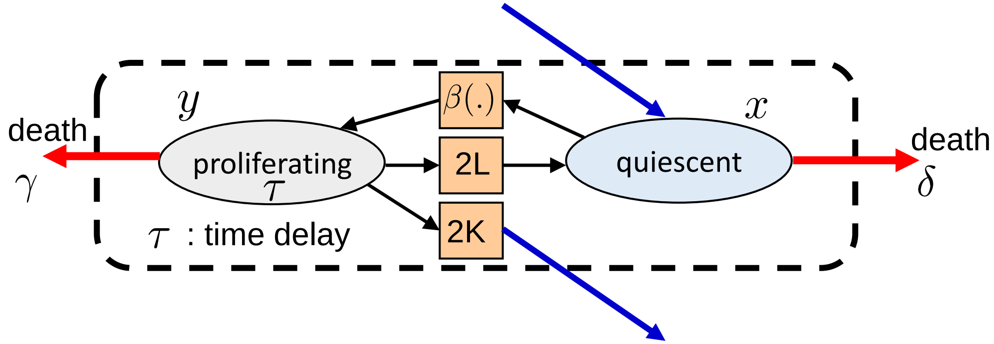
Gene Regulatory Networks





Cell Population Dynamics in AML



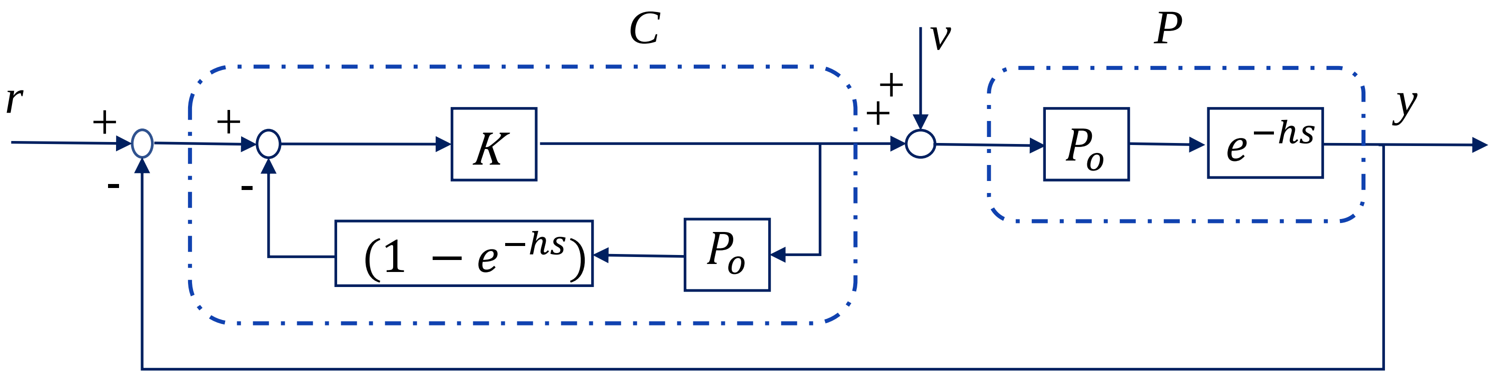


Distributed Parameter Systems

- Systems with delay (TDS)
- Systems modeled by PDEs: examples include

  flexible structures, biochemical processes, wave equation, etc.

Classical Control of Stable LTI Plants with Time Delay: The Smith Predictor



Main idea: elimination of time-delay from the feedback loop

$K(s)$ is designed to stabilize $P_0(s)$, leading to complementary sensitivity $T_0(s) = \dfrac{P_o(s)K(s)}{1 + P_o(s)K(s)}$

$$\Rightarrow \qquad T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = T_0(s)e^{-hs}$$

Issues:

- Robustness against plant uncertainty and delay mismatch?
- Above structure is not applicable for unstable plants, at least not directly.

Extension to Unstable Systems

Watanabe and Ito (1981)

Matausek and Micic (1996)

G. Meinsma and H. Zwart (2000)

L. Mirkin, N. Raskin (2003)

Q.-C. Zhong (2003)

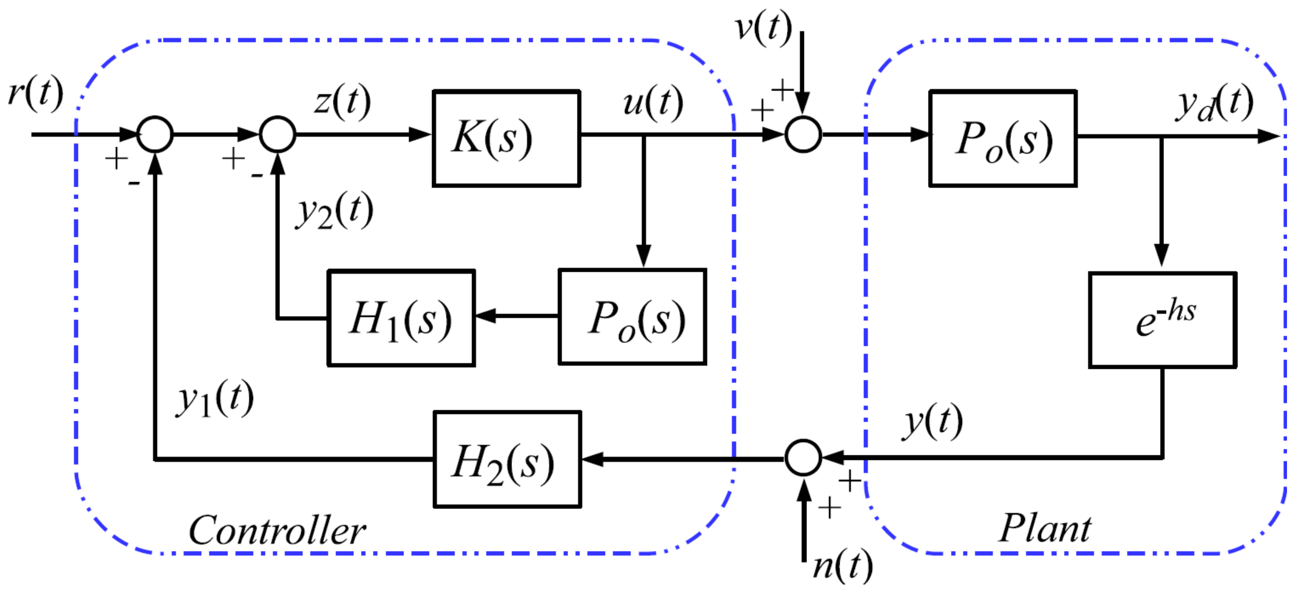
Q. Zhong and G. Weiss (2004)

J.E. Normey-Rico and E.F. Camacho (2002)

P. Garcia, P. Albertos (2013)

R. Sanz, P. Garcia and P. Albertos (2018)

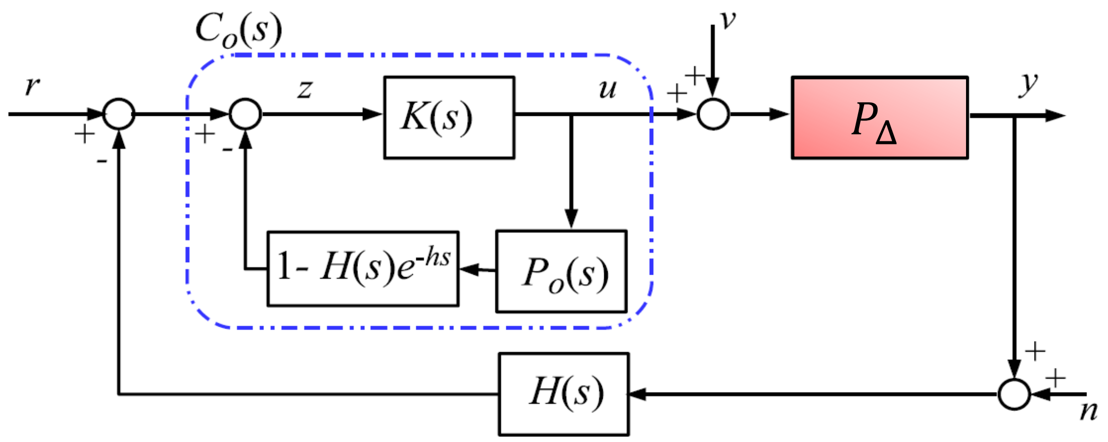
...



Main argument:
K should not depend on h, and the designs of H_1 and H_2 should be simple

Extension to Unstable Systems – Robust Design

Yegin and Ozbay, *Systems and Control Letters*, 2023



$$\mathcal{P} = \left\{P_\Delta := (1 + W_m\Delta_p)P_o e^{-sh},\ \Delta_p \text{ is stable},\ \|\Delta_p\|_\infty < 1\right\}$$

W_m is stable: uncertainty weight, captures delay mismatch as well

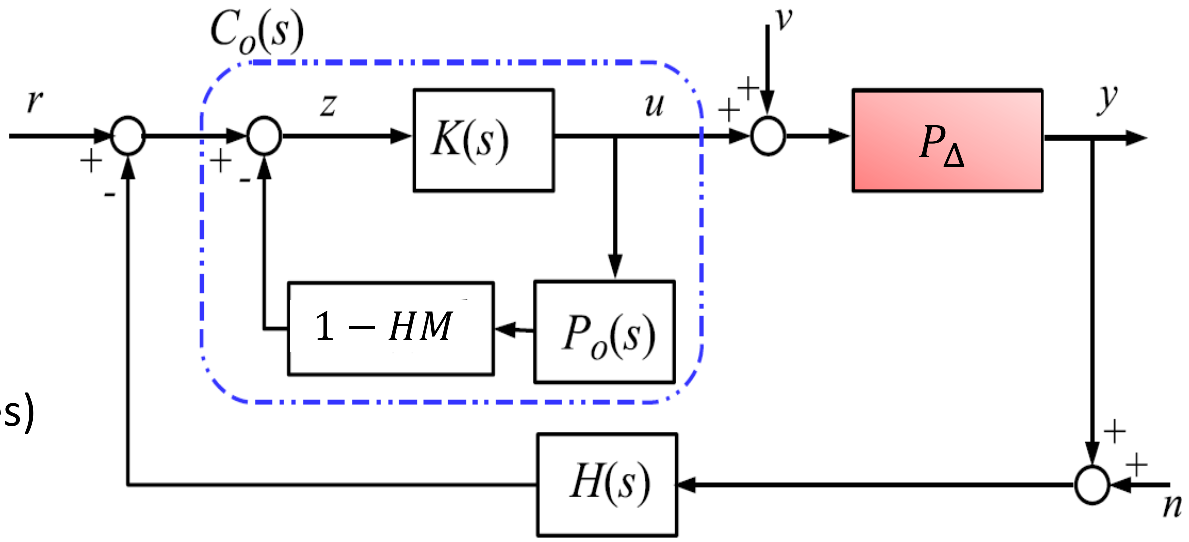
Step 1: $K(s)$ is designed to stabilize $P_0(s)$, leading to complementary sensitivity $T_0(s) = \dfrac{P_o(s)K(s)}{1+P_o(s)K(s)}$

Step 2: $H(s)$ is stable and satisfies

Nevanlinna-Pick problem: $\begin{cases} H(p_i) = e^{hp_i} \\ ||H||_\infty < 1/||W_m T_0||_\infty \end{cases}$ p_i is a pole of P_0 in RHP

When $W_m = 0$ (nominal case) $\Rightarrow$ $Y(s)/R(s) = T_0(s)e^{-hs}$

Extension to more general DPS:



$P_0(s) = N(s)/D(s)$ *and* $M(s)$ is an inner function replacing e^{-hs}

$N(s)$ is outer and $D(s)$ is rational (finitely many unstable poles)

$$\mathcal{P} = \left\{P_\Delta := (1 + W_m\Delta_p)P_o M,\ \Delta_p \text{ is stable},\ \|\Delta_p\|_\infty < 1\right\}$$

W_m is stable: uncertainty weight

Step 1: $K(s)$ is designed to stabilize $P_0(s)$, leading to complementary sensitivity $T_0(s) = \dfrac{P_o(s)K(s)}{1 + P_o(s)K(s)}$

Step 2: $H(s)$ is stable and satisfies

Nevanlinna-Pick problem: $\begin{cases} H(p_i) = 1/M(p_i) \\ ||H||_\infty < 1/||W_m T_0||_\infty \end{cases}$ $\quad p_i$ is a pole of P_0 in RHP

When $W_m = 0$ (nominal case) $\Longrightarrow$ $Y(s)/R(s) = T_0(s)M(s)$

Outline

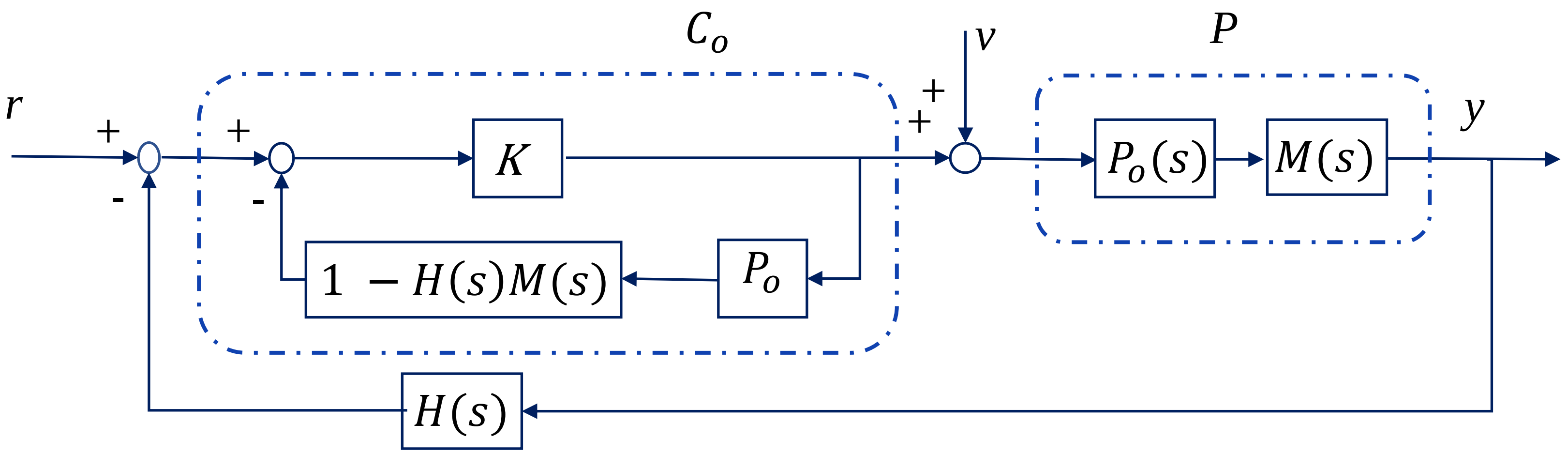
Part I

- What is strong stabilization?
- Unstable plants are “difficult” to control
- Why strong stabilization?
- Strong stabilization methods

Part II

- Distributed Parameter Systems (DPS) and Time Delay Systems (TDS)
- Strong stabilization for DPS
- Open research problems

Strong Stabilization of DPS

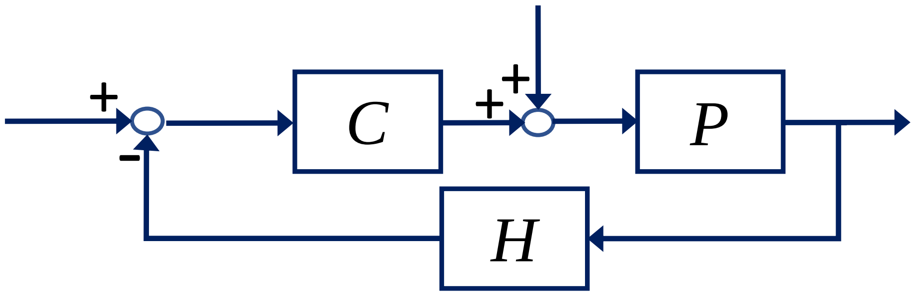


$P_0(s) = N_o(s)/D(s)$ and $M(s)$ is inner

$N_o(s)$ is outer and $D(s)$ is rational (plant has finitely many unstable poles)

Question: When is C_o stable?
Difficult problem!

Back to strong stabilization under sufficient conditions (small gain)



Assume $H(s) = 1$ and $P(s) = N(s)/D(s)$ $\quad N = MN_o$ $\quad C(s) = (U(s) - D(s))/N(s)$

From earlier discussion:

Pick an outer function W_o with $W_o(\infty) = 1$ and define $R(s)$ from $W_o D =: 1 - R$

Now try to find a unit $W_o U = W_o D + N W_o Q = 1 - (R - NQ_1)$

Then $Q = W_o^{-1} Q_1$ is a strongly stabilizing controller if $Q_1 \in \mathcal{H}^\infty$ satisfies $\|(R - NQ_1)\|_\infty < 1$

Examples will be given for the following problems

SS0: Given P find a strongly stabilizing controller C

SS1: Given P, W_1 and ρ, find a strongly stabilizing controller C such that $\|W_1 S\|_\infty \leq \rho$, $\quad S = \frac{1}{1+PC}$

Example: Strong stabilization for infinite dimensional systems

$$P(s) = \frac{(s-4)e^{-3hs}}{(s+1-2e^{-0.4s})} \begin{bmatrix} \frac{1}{s+2} & \frac{-1}{s+4} & \frac{1}{s+3} \\ 0 & 0 & \frac{e^{-hs}}{s+1+e^{-s}} \end{bmatrix}, \quad h>0 \qquad P = D^{-1} M N_o N_1$$

$$M(s) = \frac{s-4}{s+4}\, e^{-3hs} \begin{bmatrix} 1 & 0 \\ 0 & e^{-hs} \end{bmatrix}, \qquad N_o(s) = \frac{1}{s+1} I, \qquad N_1(s) = \frac{s-p}{s+1-2e^{-0.4s}} \begin{bmatrix} \frac{s+4}{s+2} & -1 & \frac{s+4}{s+3} \\ 0 & 0 & \frac{s+4}{s+1+e^{-s}} \end{bmatrix}$$

$$D(s) = \frac{s-p}{s+1} I \qquad p>0 \text{ being the only root of } s+1-2e^{-0.4s}=0 \text{ in } \mathbb{C}_+ \qquad p \approx 0.5838$$

SS0 $\qquad U = D + M N_o N_1 C \qquad C \in \mathscr{H}^\infty$

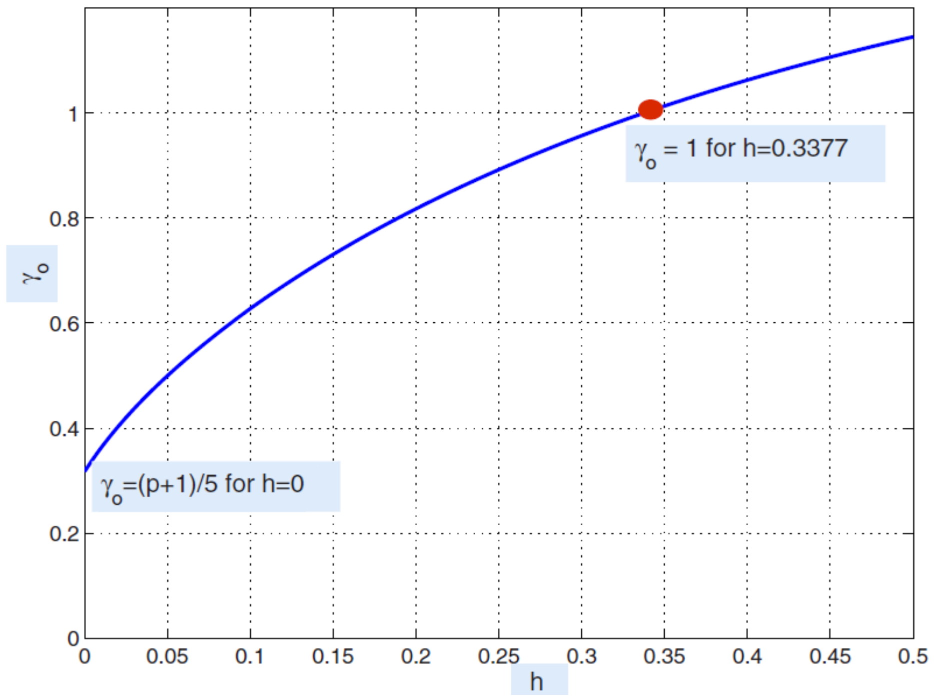
$$N_1^\dagger(s) = \frac{s+1-2e^{-0.4s}}{s-p} \begin{bmatrix} 2\frac{s+2}{s+4} & 0 \\ 1 & \frac{s+1+e^{-s}}{s+3} \\ 0 & \frac{s+1+e^{-s}}{s+4} \end{bmatrix} \in \mathscr{H}^\infty. \qquad C = N_1^\dagger C_1 \qquad C_1 \in \mathscr{H}^\infty$$

Define $R := (D - I)$, if there exists $C_1 \in \mathscr{H}^{\infty}$ satisfying $\| R + M N_o C_1 \|_{\infty} < 1$

then we have strong stabilization.

Conclusion: SS0 has a solution if the following H_{∞} control problem is solvable

$$\gamma_o := \inf_{Q \in \mathscr{H}^{\infty}} \left\| \frac{p+1}{s+1} - \frac{(s-4)}{(s+4)(s+1)} e^{-4hs} Q \right\|_{\infty} < 1.$$



This approach gives a strongly stabilizing controller if and only if the delay is "small enough"

$$h < 0.3377$$

SS1: Given P, W_1 and ρ, find a strongly stabilizing controller C such that $\|W_1 S\|_\infty \leq \rho$

Assume that $\rho > \sqrt{2}\|W_1 D\|_\infty$, then we can find $V_\rho \in \mathscr{H}^\infty$ such that $V_\rho^{-1} \in \mathscr{H}^\infty$ and

$$|V_\rho(j\omega)|^2 = \frac{1}{2} - |\rho^{-1} W_1(j\omega) D(j\omega)|^2 \qquad \omega \in \mathbb{R}.$$

SS1 is solvable if

$$\gamma_1 := \inf_{Q_1 \in \mathscr{H}^\infty} \|V_\rho^{-1} R + N Q_1\|_\infty < 1.$$

$$D(s) = \frac{s-p}{s+1}, \qquad N(s) = \frac{s-4}{(s+4)(s+1)} e^{-4hs}, \qquad \text{with } p = 0.5838 \text{ and } h > 0.$$

Take $\rho = 2$ and $W_1(s) = \frac{s+1}{10s+1}$,

h	0	0.01	0.05	0.10	0.13	0.1354	0.14	0.15	0.2
γ_1	0.45	0.52	0.71	0.89	0.98	0.9991	1.013	1.041	1.165

the largest h for which we can find a solution to SS1 using this method is 0.1354.

Open Research Problems

- H_∞ optimal controllers in the set of all strongly stabilizing controllers.
- Delay margin optimization with strongly stabilizing controllers.
- Sensitivity and mixed sensitivity minimization by stable controllers for infinite dimensional systems (including time delay systems).
- Fixed order or low order stable controllers.
- Strongly stabilizing controllers from the extended Smith Predictor.
- Extension to nonlinear systems.

Thank You

Acknowledgements: the work presented here contain material from publications with the following colleagues and former PhD students (in random order):

- Nevanlinna-Pick interpolation and applications to strong stabilization:

C. Foias, A. Tannenbaum, S. Gumussoy, M. Wakaiki, Y. Yamamoto, V. Yucesoy

- Strong stabilization for finite dimensional systems and time delay systems:

A. N. Gundes, S. Garg, O. Toker, M. Zeren, M. O. Yegin